SimonFest
California Institute of Technology
March 27 – 31, 2006

Abstracts

Critical Phenomena via Random Walk Representations: Selected Classics and Some Recent Results

Michael Aizenman
Princeton University

SimonFest offers an opportunity to reminisce about the contribution of "soft" methods in the study of critical phenomena in Statistical Mechanics—methods which have been warmly embraced by Barry Simon and significantly advanced in his work on the subject. In addition to a partial review of the results derived at that period, I shall describe also some related recent results. The latter includes a spillover of statistical mechanics insights into the analysis of Schrödinger operators, and also the recent proofs by M. Heydenreich and R.W. van der Hofstad, and of the speaker with V. Papathanakos, of a long outstanding conjecture concerning a drastic effect of the boundary conditions (periodic versus bulk/free) on the nature of the scaling limits of critical models, in particular above the upper-critical dimensions.

Barry and the Quantum Hall Effect: An Argument with the Laughlin Argument

Yosi Avron
Technion – Israel Institute of Technology

I shall review the geometric approach to the integer quantum Hall effect. In particular, I shall survey joint work with Barry Simon and Ruedi Seiler on the interpretation of the Hall conductance as an index.

Schur Processes

Alexei Borodin
California Institute of Technology

Measures on partitions with weights given by suitable products of Schur symmetric functions have recently found a variety of applications from Toeplitz determinants to representation theory, stochastic growth processes,

and random tiling models. The goal of the talk is to give a survey of known results on such measures.

A New Approach to Spectral Gap Problems

Jean Bourgain
Institute for Advanced Study, Princeton

Using combinatorial techniques such as the sum-product phenomenon in finite fields, we exhibit new classes of expanders in $SL^2(p)$ and $SU(2)$. Various applications are given, in particular, to the quaquaversal tiling problem.

Lyapunov Exponents and Spectral Analysis of Ergodic Schrödinger Operators

David Damanik
California Institute of Technology

The spectral analysis of an ergodic family of one-dimensional Schrödinger operators typically starts out with an investigation of the Lyapunov exponent of the associated energy-indexed Schrödinger cocycle over the given ergodic transformation. For example, the absolutely continuous spectrum is given by the essential closure of the set of energies for which the Lyapunov exponent vanishes. We review some general results in this context, particularly Kotani theory, and their application to concrete models.

Non-Self-Adjoint Operators and Pseudospectra

E B Davies
King's College, London

The theory of pseudospectra has grown rapidly since its emergence from within numerical analysis around 1990. We describe some of its applications to the stability theory of differential operators, to WKB analysis and even to orthogonal polynomials. Although currently more a way of looking at non-self-adjoint operators than a list of theorems, its future seems to be assured by the growing number of problems in which the ideas are clearly of relevance.

Riemann–Hilbert Methods in the Theory of Orthogonal Polynomials

Percy Deift
Courant Institute, NYU

In this talk I will describe various applications of the Riemann–Hilbert method to the theory of orthogonal polynomials on the line and on the circle.

Recent Developments in Quantum Mechanics with Magnetic Fields

Laszlo Erdös
Universität München

We present a review of recent developments concerning rigorous results on Schrödinger operators with magnetic fields. Special attention will be paid to inhomogeneous fields and many-body problems in magnetic fields.

Fitting a C^m Smooth Function to Data

Charles Fefferman
Princeton University

The talk describes an efficient algorithm to compute a C^m function whose graph passes through (or close to) N given points in R^n. The task is delicate in case the points lie near a low-degree algebraic hypersurface. This is based on joint work with Bo'az Klartag.

Two- and N-body Schrödinger Operators and Resonances: Barry Simon's Legacy (so far!)

Richard Froese
University of British Columbia

In this talk I will review Barry Simon's contributions to the mathematics of non-relativistic N particle quantum systems and to the theory of resonances.

Atomism and Quantization

Jürg Fröhlich
ETH, Zürich

I present a novel approach to understanding the mean-field limit of quantum many-body systems, in particular of atomic Bose gases. An Egorov-type theorem relates the quantum dynamics to the dynamics in the mean-field limit, which is given by a Hartree equation. In the mean-field limit, matter is described as a continuum field theory. I show how, starting from certain Hamiltonian continuum theories of matter, the passage to an atomistic description of matter can be understood as a (deformation) quantization. I then discuss some applications of the mean-field description of quantum many-body systems to the physics of stars. In particular, I will describe some recent results on stellar collapse.

N-body Quantum Scattering and Quantum Resonances: An Overview

Christian Gérard
Université Paris-Sud

We will review the progresses made in the last fifteen years in scattering theory of N-particle Hamiltonians and in resonance theory for Schrödinger operators.

Orthogonal Polynomials on the Unit Circle: From Szegő to Simon

Leonid Golinskii
Institute for Low Temperature Physics, Kharkov

The theory of orthogonal polynomials on the unit circle has experienced an unparalleled growth lately, due primarily to Barry Simon and his collaborators whose discoveries led to a substantial reevaluation of perceptions as to the nature of orthogonal polynomials on the unit circle and their applicability. We overview the evolution of the theory focusing on Simon's contribution in the area.

Geometric Scattering Theory: From Particles to Fields

Gian Michele Graf
ETH, Zürich

Scattering theory has been shaped by spectral and geometric approaches. Many among the main results in the last fifteen years or so depended on a combination of both. We shall review some of the contributions by Barry Simon (and others) to these methods, as well as some of their applications, such as to the quantum mechanical N-body problem, or to Rayleigh and Compton scattering. Some open problems will be mentioned, too.

Molecular Quantum Mechanics in the Born–Oppenheimer Limit

George Hagedorn
Virginia Tech

Born–Oppenheimer approximations describe molecular quantum mechanics in the limit of large nuclear masses. Although these approximations are almost eighty years old and fundamental to theoretical chemistry, their rigorous mathematical analysis began only thirty years ago. Most of this analysis has concentrated on validating existing physical theories, but some has led to new insights concerning molecular dynamics.

We review the mathematical work in this subject and describe some directions in which we hope some future progress might be made.

Perturbation Theory and Atomic Resonances Since Schrödinger's Time

Evans M. Harrell
Georgia Institute of Technology

Quantum theory makes a sharp distinction between bound states and scattering states, the former associated with point spectrum and the latter with continuous spectrum. Resonances associated with quasi-stationary states bridge this distinction, and have posed mathematical challenges since the beginning of the Schrödinger theory. Here the development of mathematical aspects of resonances in atomic physics is reviewed, with particular reference to the role of the Stark effect, and perturbations of bound states.

Barry Simon's Contribution to Magnetic and Electric Fields and the Semiclassical Limit

Ira W. Herbst
University of Virginia

In this talk I will review some of Barry Simon's seminal contributions to the theory of Schrödinger operators with external magnetic and electric fields. I will also talk about a beautiful theorem of Barry's on the energy splitting in multidimensional double wells in the large coupling limit.

Bound State Problems in Quantum Mechanics

Dirk Hundertmark
University of Illinois at Urbana-Champaign

We give a review of semi-classical estimates for bound states and their eigenvalues for Schrödinger operators. Motivated by the classical results, we discuss their recent improvements for single particle Schrödinger operators as well as some applications of these semi-classical bounds to multi-particle systems, in particular, large atoms and the stability of matter.

Fermi Golden Rule in Quantum Statistical Mechanics

Vojkan Jaksic
McGill University

The mathematical foundations of time-dependent perturbation theory were laid down by Barry Simon in his seminal 1973 Annals paper. In this talk I will discuss the time-dependent perturbation theory and Fermi Golden Rule in the context of quantum statistical mechanics. The talk will be of a review nature and I will focus on the historical aspects as well as the applications to the problem of return to equilibrium (zeroth law of thermodynamics). This talk is based on joint work with C.-A. Pillet.

Ergodic Schrödinger Operators: Recent Advances

Svetlana Jitomirskaya
University of California, Irvine

We will review recent progress on the spectral theory of quasiperiodic operators and on the Anderson model.

Orthogonal Polynomials: First Minutes

Sergey Khrushchev
Atilim University, Ankara, Turkey

It is standard to refer to Chebychev, Gauss and Jacobi as the creators of Orthogonal Polynomials. In fact, this topic goes back to the very early times of analysis, namely to March of 1655 when Wallis completed his famous book "Arithmetica of Infinitorum." This book contained a remarkable Section 191, in which Wallis presented his understanding of a solution to the functional equation $b(s)b(s+2) = (s+1)^2$ found by Brouncker. Wallis' presentation was not very clear and posed questions on Brouncker's proof rather than explaining it. Later in his main paper on Continued Fractions (1739), Euler paid great attention to this result of Brouncker and mentioned that it would be highly desirable to recover Brouncker's original arguments. In this talk, we present such a recovery and show how this problem is related to orthogonal polynomials.

Some Sum Rules

Rowan Killip
UCLA

I will describe some examples of sum rules—simple equations relating the coefficients of operators to their spectral data—and outline their applications in forward/inverse spectral analysis.

Imbedded Singular Spectrum for Schrödinger Operators

Alexander Kiselev
University of Wisconsin at Madison

We will review recent results on the imbedded singular spectrum. This will include examples with a dense set of imbedded eigenvalues, and examples where wave operators exist but are not asymptotically complete due to the presence of a singular continuous spectrum. We will also discuss estimates on the size of the set where the singular spectrum may be supported, which can be thought of as nonlinear versions of well-known estimates for the Fourier transform.

Exotic Spectra: A Review of Barry Simon's Central Contributions

Yoram Last
The Hebrew University of Jerusalem

The talk will review some of Barry Simon's central contributions concerning what he termed exotic spectral properties. These include phenomena such as Cantor spectrum, thick point spectrum, and singular continuous spectrum.

Selberg's Zeta Function and Its Children

Peter Perry
University of Kentucky at Lexington

Ideas of quantum-mechanical scattering theory have been applied with remarkable success to study the spectrum of the Laplacian on complete, non-compact Riemannian manifolds with "simple geometry at infinity" and clarify its connection with classical mechanics, that is, geodesic flow. This talk reviews recent developments in scattering theory for Riemannian manifolds with constant curvature at infinity, including exact trace formulas, connections with Selberg's zeta function for geodesic flow, and the inverse resonance problem.

Complex Scaling in Atomic Physics: In and Out of External Fields

William P. Reinhardt
University of Washington at Seattle

Applications of complex scaling in computational atomic and molecular physics began almost immediately as the mathematical theory developed: life-times of doubly excited states of two-electron atoms starting things off. Quite quickly, computational experiments indicated that similar theory (as will be discussed later in the week by Herbst) must exist for the problems of atomic structure in both constant, and oscillatory external fields. Applications abound in these areas, and early computational developments and examples will be over-viewed. Finally, more recent, and highly successful, use of "exterior" complex scaling has allowed the Holy Grail of computational atomic theory to be ever more closely approached: the three-body Coulomb scattering problem at zero energy will be reviewed, and recent progress seen to have been quite remarkable.

Barry Simon's Contributions to Quantum Field Theory

Lon Rosen
University of British Columbia

Beginning with a brief history of constructive quantum field theory, we present a review of Barry Simon's contributions to the field. We emphasize his role in advocating Euclidean methods, his introducing ideas and techniques from statistical mechanics such as correlation inequalities and the Lee–Yang Theorem, and his developing functional analytic methods such as hypercontractivity.

On the Formation of Gaps in the Spectrum of Schrödinger Operators with Quasi-Periodic Potentials

Wilhelm Schlag
California Institute of Technology

We will review some work by Michael Goldstein and the speaker on the formation of gaps in the spectrum of operators with quasi-periodic potentials defined by the shift on the circle. We will assume only positive Lyapunov exponents and results will be obtained for almost every value of the angle of the shift. The methods will involve large deviation theorems, the avalanche principle, as well as resultants. Particular emphasis will be given to a quantitative separation property of the eigenvalues.

Orthogonal Polynomials from Jacobi to Simon

Vilmos Totik
University of South Florida and University of Szeged, Hungary

We give an overview of some questions in the theory of orthogonal polynomials, from classical ones to some very recent progress. Orthogonal polynomials on the unit circle, a favourite of Barry Simon, will be discussed in the accompanying talk by Leonid Golinskii.