Spectral Theory of Differential Operators
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SPECTRAL ANALYSIS OF MULTIPARTICLE SCHRÖDINGER OPERATORS

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The first lecture is an introduction to some recent work by Peter Perry, Israel Sigal and me [2,3] on the spectral analysis of N-body Schrödinger operators. Our work is based in part on some beautiful ideas of Eric Mourre [1].

Given masses m_j and functions (potentials) on R^ν, V_{ij}, with $1 \le i < j \le N$, we define an operator H on $L^2(R^{\nu(N-1)})$, as follows: think of $R^{\nu(N-1)}$ as N tuples of vectors r_j in R^ν with $\sum_1^N m_j r_j = 0$. Let $V = \sum_{i<j} V_{ij}\,(r_i - r_j)$ and let H_0 be the Laplace Beltrami operator associated to the metric $\sum m_j dr_j^2$. Then $H = H_0 + V$.

Perry, Sigal and Simon consider potentials $V_{ij} = V_{ij}^{(1)} + V_{ij}^{(2)} + V_{ij}^{(3)}$ where the following six operators are $-\Delta$-compact on $L^2(R^\nu)$: (1) $(1+|x|^2)V^{(1)}$; (2) $(1+|x|)V^{(2)}$; (3) $(1+|x|)^2\nabla V^{(2)}$; (4) $V^{(3)}$; (5) $(1+|x|)\nabla V^{(3)}$; (6) $(1+|x|)^2\nabla\nabla V^{(3)}$. Roughly speaking any $x^{-2-\varepsilon}$ potential is allowed; slower falloff requires more smoothness but very slow falloff (e.g. $(\ell n r)^{-1-\varepsilon}$) is allowed.

<u>Theorem [2,3]</u> Under the above conditions:

(i) H has empty singular continuous spectrum.

(ii) The thresholds of H are a closed countable set.

(iii) Non-threshold eigenvalues are of finite multiplicity and such eigenvalues can only accumulate at thresholds.

References

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SCHRÖDINGER OPERATORS WITH ALMOST PERIODIC POTENTIALS

In the second lecture some general conjectures and results about operators of the form

$$-d/dx^2 + V(x) = H$$

on $L^2(-\infty,\infty)$, where V is a (Bohr) almost periodic function, are discussed. This is a subject of intense current interest [1,2,4,5,9]. Earlier significant results can be found in [3,6,7,8].

Two main features are to be expected:

(i) The spectrum of H is a Cantor set for "most" almost periodic V.

(ii) If V is multiplied by a sufficiently large constant, H will have dense point spectrum at low energies.

Connected with (i) is anomalous long time behavior for the quantity $(\phi, \exp(-itH)\phi)$ [1]. So far the proven results concerning (i) and (ii) are somewhat limited: (i) is proven for generic limit periodic V [1,5], and (ii) has been announced [2] for some special finite difference analogs of H. Sarnak [9] has proven (ii) for such operators with V a special complex valued function.

One interesting application is to think of H as a Hill operator (linear stability operator in classical mechanics) as would arise in the study of the rings of Saturn. [1].

References

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