DIFFERENTIAL EQUATIONS
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m-Functions and the Absolutely Continuous Spectrum of One Dimensional Almost Periodic Schrödinger Operators

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Abstract. We describe a program initiated by Moser [1], and Johnson-Moser [2], and then developed by Kotani [3] (Kotani's work was extended to the discrete case by Simon [4]) and Deift-Simon [5]. We discuss relations between the density of states, the Lyaponov exponent, and the classical m-function of Weyl. In particular, we obtain Kotani's results that the essential support of the absolutely continuous spectrum is precisely the set where the Lyaponov exponent vanishes, and that in the random non-deterministic case the Lyaponov exponent is a.e. positive. We also describe the Deift-Simon results that, in the discrete case, the Lebesgue measure of the set where the Lyaponov exponent vanishes is at most 4, and the construction of continuum eigenfunctions for the absolutely continuous spectrum.

References

1. J. Moser, An example of a Schrödinger operator with almost periodic potential and nowhere dense spectrum, Comm. Math. Helv. 56 (1981), 198-224.

2. R. Johnson and J. Moser, The rotation number for almost periodic potentials, Comm. Math. Phys. 84 (1982), 403-438.

3. S. Kotani, Lyaponov indices determine absolutely continuous spectra of stationary random one-dimensional Schrödinger operators, Proc. Kyoto Stoch. Conf., 1982.

4. B. Simon, Kotani theory for one dimensional stochastic Jacobi matrices, to appear in Comm. Math. Phys.

5. P. Deift and B. Simon, Almost periodic Schrödinger operators, III. The absolutely continuous spectrum in one dimension, to be submitted to Comm. Math. Phys.