Schrödinger Operators with Random and Almost Periodic Potentials

B. Simon

Division of Physics, Mathematics and Astronomy,
California Institute of Technology, Pasadena, CA 91125, USA

Schrödinger operators and their discrete analogs were discussed especially in one space dimension. These operators were of the form

$$H = H_0 + V$$

where V is the sample function of an ergodic process. Examples include almost periodic functions and random processes. A particularly simple random example occurs in the discrete case where one can take V to be independent, identically distributed random variables with distribution $d\kappa$. This is known as the Anderson model. Among the topics discussed were:

1. The basic objects of the theory including the integrated density of states, k(E), the transfer matrix and the Lyaponov exponent, $\gamma(E)$. The Thouless formula

$$\gamma(E) = \int_{-\infty}^{\infty} \ln |E - E'| \, dk(E')$$

relates them.

2. Localization for the Anderson model in one dimension, that is the tendency for this model to have dense point spectrum with exponentially decaying eigenfunctions. We followed the approach of Kotani, Delyon, Levy, Souillard, Simon and Wolff.

3. Kotani theory, that is, a set of ideas relating the m function of Weyl theory, the Lyaponov exponent and the absolutely continuous spectrum. Two results of this theory are that $\{E | \gamma(E) = 0\}$ is the essential support of the absolutely continuous spectrum, and that this set is empty if the process is deterministic.

4. The Maryland model, that is, the discrete Schrödinger operator with potential

$$V(n) = \lambda \tan(\pi \alpha n + \theta)$$

This model has a computable density of states and spectral properties which depend on the Diophantine properties of α. In particular, one can find Hamiltonians with identical densities of states so that one has pure point spectrum and the other purely singular continuous spectrum.

References

1. H.L. Cycon, R.G. Froese, W. Kirsch and B. Simon: Schrödinger Operators with Application to Quantum Mechanics and Global Geometry, Springer 1987
2. R. Carmona: Random Schrödinger Operators, in Summer School of Probability of St. Flour XIV, 1984, Lecture Notes in Mathematics, vol. 1180, Springer-Verlag
3. T. Spencer: The Schrödinger Operator with a Random Potential: A Mathematical Review, in Critical Phenomena, Random Systems, Gauge Theories, Les Houches XLIII.