Remembrances of Derek William Robinson, June 25, 1935–August 31, 2021

*Editors: Michael Barnsley, Bruno Nachtergaele, and Barry Simon*
Contributors: Alain Connes, David Evans, Giovanni Gallavotti, Sheldon Glashow, Arthur Jaffe, Palle Jorgensen, Aki Kishimoto, Elliott Lieb, Heide Narnhofer, David Ruelle, Mary Beth Ruskai, Adam Sikora, and A. F. M. ter Elst

Introduction by the Editors

Derek William Robinson (June 25, 1935–August 31, 2021) was a founding figure in the modern mathematical approach to the foundations of statistical mechanics, together with Roland Dobrushin, Rudolph Haag, Oscar Lanford, and especially David Ruelle. With Ola Bratteli, he wrote what has long been the standard monograph on the use of operator algebra methods in quantum statistical mechanics [2] and an important resource for those studying C^*-algebras, an area in which he likewise made significant contributions. Later he worked on analysis on Lie Groups where he also wrote two monographs [4, 17].

He was born in Southern England; his father was from rural Northern Ireland. He entered Oxford in 1954 and graduated with honors in mathematics in 1957. Because

the head tutor in maths at Wadham College was an applied mathematician, Derek wound up choosing hydrodynamics and quantum mechanics as his special subjects. He was offered a graduate scholarship in physics at Oxford even though, as he commented, *the only openings they had in theoretical physics were in nuclear physics, down and dirty stuff, so I ended up having to work through this stuff in which I had absolutely no interest.*
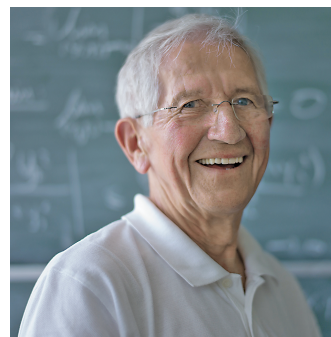
Figure 1. Derek Robinson (1935–2021).

Although his PhD thesis was in nuclear theory, he mainly studied quantum field theory and particle physics. His directions were heavily influenced by summer schools he attended in those subjects in Naples and Edinburgh, where he met and became good friends with two future Nobel Laureates,

Tini Veltman and Shelly Glashow. Indeed, Veltman was best man at Derek's wedding and the Glashows spent part of their honeymoon in the Robinson's home in the South of France.

One of the lecturers in the school in Naples was Res Jost who was Pauli's successor at the ETH in Zurich. Derek was so struck by his research that he took a NATO postdoc in Jost's group in 1960–1962 immediately after he finished his PhD. He was allowed to take this fellowship at ETH even though Switzerland is not a NATO country! One of the other postdocs in Jost's group was Ruelle, and a postdoctoral visitor was Araki, who lectured on von Neumann algebras.

Derek's second postdoctoral mentor was Rudolph Haag who together with Kastler invented the C^*-algebraic approach to quantum field theory and to quantum statistical mechanics. Derek worked with Haag for roughly four years (1962–1966), two years at the University of Illinois where Haag was a professor, then 18 months in Munich while Haag was on leave. When Haag returned to Illinois at the end of 1965, Derek stayed in Europe spending three months at IHES at Ruelle's invitation and six months in Marseille as Kastler's visitor.

Robinson then moved to a position within the theory group at CERN during 1966–1968. While the group mainly focused on high energy physics, during this period, Derek primarily did statistical mechanics (as he had during his visits to IHES and Marseille) doing seminal work that we'll describe below. This work led to several tenure offers. While in Geneva, Derek met his wife Marion, the daughter of a diplomat from New Zealand who had spent her youth in Canada and Geneva.

Derek and Marion decided to accept the offer Kastler had arranged for a professorship in theoretical physics from the University of Aix-Marseille. One of the results of the 1968 student uprising was to make it possible for non-French people to have permanent public service jobs including permanent university appointments. Derek was one of the first non-native French professors! Derek and Marion built a house in Bandol, a town on the Mediterranean coast east of Marseille where the Kastlers lived.

During his time in Marseille, Derek's research shifted more toward problems in the theory of operator algebras and he began working with Ola Bratteli with whom he'd work for over 40 years. This included their book, which they initially planned to be 300–400 pages but turned out to be two volumes totaling more than 1050 pages!

In early 1976, Angas Hurst (the H of the GHS inequalities) lured Derek to Adelaide to lecture in the "Summer" Research school. Derek was taken with Australia and he and Marion began thinking about a permanent move there. He applied for a professorship in pure mathematics

Figure 2. Derek and Marion in Bandol.

Figure 3. The house in Bandol.

at the University of New South Wales where he was from 1978 until 1982. Derek then moved to a professorship at the Institute For Advanced Studies at the Australian National University in Canberra, a position he held for 19 years until his retirement. While he became emeritus in 2000, he continued to have a research grant and wrote papers until shortly before his death.

Derek was a serious competitive cyclist and at one point he wrote that *you don't necessarily peak as a cyclist when you are young, I found I was faster at time trials at the age of 50 than I had been as an eighteen-year-old. In 2002 I was very pleased to win the World Masters Games 20k cycling race in my age range. This was a time trial and I was the last person*

to start as I was the top seed and I completed the course in a little over 32 minutes. In some ways I consider this my greatest achievement!

We turn to a description of some of Derek's best known research. In the mid-1960s, there was a significant paradigm shift in which Derek was a major figure so that the study of systems in infinite volume took over and new mathematical realizations, problems, and objects came to the fore. There was considerable resistance in the theoretical physics community even though currently the picture is so accepted that it is hard to realize there was ever an issue. Derek's work focused on issues special to the quantum case, although his paper with Ruelle on entropy [20] in the classical setting was a factor in the classical framework which Derek sought to extend to quantum systems.

One of his key discoveries concerned the invention of asymptotically abelian actions. For classical systems, an important property of infinite volume equilibrium states is the uniqueness of decomposition into pure phases—in mathematical terms, the crucial underlying fact is that the translation invariant probability measures are a Choquet simplex whose extreme points are the ergodic measures. It inherits this from the fact that all probability measures on a compact Hausdorff space are a simplex.

For quantum systems, there is an issue in extending this notion: the set of all states on a C^* algebra is a simplex if and only if the algebra is abelian and, of course, general quantum systems are nonabelian! Derek discovered the idea of an asymptotically abelian action by translations — those with the property that for any pair, A, B of operators, A and the distant translates of B asymptotically commute. The set of states invariant under such an action are a simplex! This idea was developed by Derek in three papers with Doplicher and Kastler [3] and one with Ruelle.

A second foundational question is the existence of an infinite volume limit for dynamics. While dynamics plays a minor role in classical spin systems, it is central to quantum spin systems, for example, via the KMS boundary condition. For both cases, one is mainly interested in interactions with an upper bound on the number of bodies involved in the potentials but since one wants arbitrarily many bodies and it is technically very convenient to have

Figure 4. Derek with his cycling medal, 2002.

a Banach space. What Derek found, as part of a series of three papers setting the framework for quantum spin systems [14], was a smaller Banach space than for the classical case where one could always define dynamics.

A third component involved finding to what extent the entropy ideas that he and Ruelle had studied for classical systems [20] extended to quantum systems. His classic paper with Lanford [8] settled many questions but also raised others including the question of strong subadditivity that we'll hear about in the contributions below of Lieb and Ruskai.

We should mention one other item before leaving statistical mechanics: the bound on speed of propagation in quantum spin systems which Derek and Elliott Lieb found [10] with the following remarkable statistic: during its first 30 years, it averaged fewer than one citation per year, but after Hastings [5] discovered its relevance to the then fledgling field of quantum information theory in 2004, there was change, so much so that recently it has averaged about three citations per *week*. The contribution of Nachtergaele below discusses this further.

Figure 5. Derek on his bike.

Figure 6. Derek with Ola Bratteli.

If there is a leitmotif to Derek's later work in C^*-algebras and the theory of elliptic operators on Lie groups (discussed further by ter Elst), it is dynamics, a subject which Derek addressed for quantum spin system in [14]. With Bratteli [1], he explored the theory of unbounded derivations on operator algebras. Derek and Ola made many striking discoveries starting with the fact that such derivations are not always closable. Derek wrote about his collaboration with Ola Bratteli in the *Notices* memorial to Bratteli [18].

For more than fifty years, Derek produced beautiful and significant mathematics.

The editors would like to thank Palle Jorgensen and Beth Ruskai for help in planning and Louisa Barnsley and Marion Robinson for information about Derek's life. We relied on some autobiographical notes of Derek's that Louisa transcribed. We expect to place them with a final draft of this article on the arXiv.

Michael Barnsley

I first noticed Derek maybe fifteen years ago. He was at the back of the tearoom of the Mathematical Sciences Institute (MSI) at Australian National University (ANU), writing neatly on the blackboard, showing mathematical things to Adam Sikora. Teas were on Thursdays before the weekly colloquium; many mathematicians, some famous, flowed in and out, leaving formulas on the boards and sometimes an empty wine bottle, marking the timelessness of mathematics and, it seems to me, of Derek. In 2014, he presented an invited paper at a conference, New Directions in Fractal Geometry; his talk concerned uniqueness of solutions to diffusion equations in media with fractal boundaries, and was illustrated by Louisa Barnsley. Over the following years, he wrote a number of papers in this direction. And over the years, Louisa and I learnt over cups of tea and biscuits with Derek in my office, of his days growing up in England, O-levels, A-levels, Oxford Entrance exam, and how he followed his own path.

In the years Louisa and I knew him, in addition to his steady mathematical work, Derek was proud of being the fastest speed walker in his age group, competing up and down Mt. Anslie, near ANU. But his real interest was mathematics, and he tracked the growing list of citations to his earlier papers, talked about his current work, and sometimes asked questions about fractal geometry, such as how best to understand the relationship between the inside and the outside of a Koch curve. He had answered his own question the next morning, before I could hazard a description.

Michael Barnsley is a professor emeritus at the Mathematical Sciences Institute, Australian National University. His email address is michael.barnsley@anu.edu.au.

Figure 7. Derek as a speed walker.

At the end, we went to his daughter's beautiful apartment in Kingston, where we were greeted by Marion and Sasha with homemade cake and tea. We sat beside Derek in his special hospital bed, installed next to huge plate glass windows, looking across Lake Burley Griffin at cows in long grasses and rushes. The sunlight streaked in. Over a number of visits, sometimes pushing him in a wheelchair beside the lovely lake, he told us the story of his early days. I told him I thought the *Notices* would publish a memorial article about him. We organized a one-day conference entitled "The Mathematical World of Derek Robinson" held on July 30, 2021. The speakers included Alain Connes, Elliott Lieb, David Ruelle, Mary Beth Ruskai, Bruno Nachtergaele, Tom ter Elst, and Juha Lehrbeck, as well as local members of the MSI. Derek had never met Juha, but was thrilled to see him on Zoom at the meeting. In his penultimate paper [19], Derek included a little fractal picture.
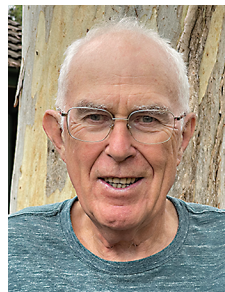

Michael Barnsley

Alain Connes

This short text will try to give an idea of Derek in Bandol in the seventies, where Daniel Kastler had gathered around his villa a number of theoretical physicists devoted to quantum statistical mechanics and algebraic quantum field theory. Derek Robinson played a key role among them by his remarkable mathematical and physical intuition, plus a conjunction of great expertise in functional analysis and deep knowledge of quantum physics.

Alain Connes is a professor emeritus at the College de France, where he is the chair of Analysis and Geometry, and at IHES, where he is the Léon Motchane chair. His email address is alain@connes.org.

Derek had decided to have his house in an isolated location not far from Daniel's house but not easily accessible by car. Besides his shining scientific talent, Derek was a joyful person with lots of wit, and I remember a dinner at his house for Thanksgiving, when at some point Derek disappeared from the dinner table, went to the kitchen, and scared all of us with a loud scream. When asked by his shivering wife why he shouted like that, Derek answered: *There is a bird in the oven. . . .*

At some point Derek decided to build a swimming pool by himself next to his house. After building a high wall, it turned out that by accident the wall fell on him! He was alone at that point and managed, with a broken back, to crawl to a phone, and call an ambulance which then took him through his bumpy road to the next hospital. After a few months in a cast Derek reappeared and his first words to me were: *I just spent months in the unit ball of a Banach space*.
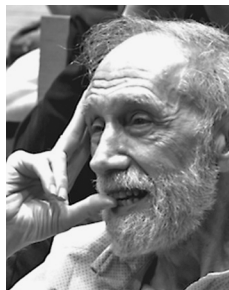
Alain Connes

David Evans

When Derek was getting settled at ANU, he invited me to visit as we had mutual interests in dynamical semigroups of positive maps on operator algebras. Consequently during a sabbatical from 1982 to 1983, I went to Canberra for four months in late 1982, to the Research School of Mathematics, which then shared a building with the Research School of Theoretical Physics. Charles Batty and Marinus Winnink were also visiting. We all met most mornings in Derek's office to discuss not only dynamical semigroups, but a variety of topics including the C^*-operator algebra approach to the Ising model which Winnink and I had worked on separately with John Lewis at Dublin. At morning coffee one day, Rodney Baxter brought a copy of his book *Exactly Solved Models in Statistical Mechanics* fresh from the press. I did not then realize what a profound impact this book would have on my future work and that it was related to a preprint I was carrying with me which

David Evans is an Honorary Distinguished Professor at Cardiff University. His email address is evansde@cardiff.ac.uk.

Vaughan Jones gave me around the same time on his new index of subfactors. The 1982 visit was the start of numerous bilateral visits between Canberra and Warwick. This initial visit to ANU was also life changing for me in that I met Pornsawan, who would become my wife.

Figure 8. Lysebu, Oslo, 2017. From left Aki Kishimoto, Palle Jorgensen, Derek Robinson, Tone Bratteli (Ola's sister), George Elliott, Reiko Kishimoto, and David Evans.

After Canberra, I visited Huzihiro Araki for few months in 1982–1983. We wrote a paper on understanding the classical two-dimensional Ising phase transition through the ground states on the one-dimensional quantum system obtained from the transfer matrices and started thinking about extending this approach to the Potts model. At the end of that sabbatical year, when visiting Winnink in September 1983, I realized that the algebraic relations of the Temperley–Lieb projections within the transfer matrix formalism for the Potts model, which I found in Baxter's book, were exactly the same as in the family of Jones projections in his tower of subfactors. Moreover the Pimsner–Popa representation of these projections, which I had just learnt about at a conference in Romania in August 1983, was the same as that discovered by Temperley–Lieb when showing an equivalence between the Potts model and an ice-type model. This and subsequent developments were triggered by Derek's invitation to Canberra. On one visit to Warwick in 1984, Derek bought a high-performance racing bike. On his next visit, to the 1986–1987 Warwick Symposium I ran on Operator Algebras, the company arranged for a car to pick him up at Heathrow airport to bring him to the store for his next cycle purchase. Derek was an integral part of that Symposium, and he left an indelible memory not only through his scientific contributions but also when turning up at the department in lycra. Derek's friendship, sense of humor, and encouragement to discuss mathematical physics on a broad spectrum are memories I treasure.

David Evans

Giovanni Gallavotti

I have been greatly influenced by Derek Robinson's works and the ideas that he developed at IHES from 1966 to 1968, which I conveyed to my students and collaborators in the early 1970s, [3, 14, 20]. I remain very grateful for his kindness and patience.

My recollection of Derek goes back to IHES in 1967 when, together with Salvador Miracle-Solé, we had the exciting experience of realizing the high temperature analyticity of the Ising model thermodynamic properties. Shortly afterward, the same idea was applied to the Heisenberg model, making use of the path integral representation found by Jean Ginibre.

Figure 9. Derek with his bike, 1995.

In 1976, after he moved to Marseille I had a brief but intense interaction with him. He and Ola Bratteli had spotted a serious mistake in a preprint by Mario Pulvirenti and me about the classical version of the KMS condition, and we had to work very hard to find a correction, which eventually was included in the fundamental book by Bratteli and Robinson [2].

Shortly afterward Derek moved to Australia. I met him again briefly, in 1997 in Brisbane at the XIIth International Congress of Mathematical Physics. He had not changed much and he confirmed that he was still cycling since his (professional level) passion for this sport had not dwindled. I could only regret that the physical distance between Italy and Australia is too large to allow us to resume the intense collaboration of 1967 and the intense interaction of 1976.

Giovanni Gallavotti

Sheldon Glashow

Derek was my close friend for decades. Soon after he moved to Australia, he told me his only regret was not doing so earlier. We overlapped throughout my two European sabbatical semesters, one in Geneva in the 60s, the other in Marseille in the 70s. We never collaborated scientifically, but we thoroughly enjoyed the release of *Sgt. Pepper's Lonely Hearts Club Band* in 1967. We have only once or twice seen each other since his continental shift. Let me recall a couple of ancient anecdotes, possibly a bit colored by time.

Derek and Marion had been an item for some time before they chose to marry. Living in France, they found it very difficult to collect the required documents. Appealing to the British consul in Marseille, they learned they needed no further documents. Because they were Commonwealth citizens, the consul could marry them there and then. And so he did, throwing in the champagne.

Upon one of Derek's visits to the States in early 1972, I invited him to dinner at the home of my then girlfriend. After the gourmet dinner she prepared, Derek and I enjoyed every drop of the Marc de Provence he had brought me from France. He collapsed on the couch for the night, while I proposed to Joan. We wed a few months later. This year, Joan and I celebrated our golden wedding anniversary.

We spent our honeymoon weeks partly in Paris, but mostly with Derek and Marion in the country house they built. They were still haunted by the French bureaucracy. Just as their new kitchen was completed, the rules governing utility dimensions were changed and their new dishwasher could not be installed. Nonetheless, Joan and I had a marvelous honeymoon ensconced with our dear friends deep in the woods surrounding beautiful downtown

Giovanni Gallavotti is a professor emeritus at Università "La Sapienza", Roma. His email address is giovanni.gallavotti@roma1.infn.it.

Sheldon Glashow is Higgins Professor of Physics emeritus at Harvard University. His email address is slg@bu.edu.

Bandol, where the wine was cheap but good. Today, it has become expensive but excellent. Plus ca change.

Derek Robinson will always remain a part me. He was a remarkable scientist and the dearest of friends.

Sheldon Glashow

Arthur Jaffe

Encouraged by my fellow student Gavin Wraith at Cambridge, UK, I travelled in the summer of 1961 as an "observer" to a school in theoretical physics held in Herceg Novi, a beautiful city on the Adriatic coast, then part of Yugoslavia. While I was too young to attend the school officially, it was serendipity to be present, for in Herceg Novi I met three students from the ETH: Derek, Klaus Hepp, and David Ruelle. Memorable teachers who interacted with everyone included Kurt Symanzik, Tulio Regge, Walter Thirring, Maurice Jacob, and André Martin. I am sure that ebullient Derek got to know them all.

Derek and I overlapped again at the meeting organized by Dick Kadison at Louisiana State University. To everyone's dismay, the Japanese mathematician Minoru Tomita arrived to announce his astounding insight, now known as modular theory. This marked the 1967 Baton Rouge meeting as one of the most influential mathematical conferences ever held in the US. I am unsure whether anyone understood Tomita's talk at the time, but Masamichi Takesaki was fascinated by it and spent the next years as a postdoctoral fellow with Kadison in Pennsylvania—working out the details and writing his famous 1970 book. My friend Sergio Doplicher (whom I met when we were both students at the IHES during the 1963–64 special year on mathematical quantum field theory) also came to Baton Rouge. Derek, Sergio, and I spent one day together after the meeting, mostly as tourists in New Orleans, when I took the Polaroid in Figure 10.

In 1979–1981, Derek and Ola Bratteli published their monumental two-volume monograph, *Operator Algebras*

Arthur Jaffe is the Landon T. Clay Professor of Mathematics and Theoretical Science at Harvard University and a past president of the American Mathematical Society. His email address is arthur_jaffe@harvard.edu.

Figure 10. Derek with Sergio Doplicher, New Orleans, after the 1967 Baton Rouge Conference.

and Quantum Statistical Mechanics. Here they give a beautiful derivation of the Tomita–Takesaki theory, along with explaining its importance and relation to other subjects, including its relation to the Kubo–Martin–Schwinger condition for Gibbs expectations and Connes's classification of factors. And today, Sergio's student Roberto Longo has become one of the leading experts on modular theory.

I believe that my next encounter with Derek came in 1968, when he visited the ETH Seminar for Theoretical Physics during the summer, in the middle of my ten-week stay there as a guest professor. The high point of the afternoon at the friendly Hochstrasse 60, Zurich address involved discussions around a table filled with cake and tea, organized by the institute secretary, Fraulein Rosemarie Hintermann. On these occasions, Derek held forth with insight and his infectious humor, and as a result we renewed our friendship.

Derek and Marion moved to Bandol when Derek joined the group of Daniel Kastler. I was fortunate to have a long visit, living in an apartment in Cassis. I had interesting excursions to their home, and enjoyed the terrace, including one evening with Derek's friend Tini Veltman.

Derek visited Harvard in 1971–1972 as my guest and the guest of Shelly Glashow and Sidney Coleman. The three had taught together in a school organized by Feza Gursey in Istanbul, and Shelly was a friend of Derek at CERN. It was in Cambridge, Massachusetts that Derek collaborated on his famous "Lieb–Robinson bound." Their result is fascinating, as it can be regarded as an analog of the finite propagation speed that Glimm and I had just shown for the two-dimensional relativistic quantum fields that we had constructed.

Figure 11. Derek and his family at his home in Canberra, January 1982.

Figure 12. Derek and his family at his home in Canberra, 1987.

After Derek left Marseille for Canberra, he made a tremendous effort to help build up mathematical physics in Australia. Derek hosted me twice in Canberra. My first trip "down under" revolved around a January 1982 summer school. Elliott Lieb and I travelled there together, and afterward, Derek organized a grand tour around Australia for the two of us. During the hospitable time in Canberra, I got to visit Derek, Marion, and their two girls frequently. My second visit to Canberra in 1987 followed a talk I gave

at the annual meeting of the Australian Mathematical Society. The photos from these nice occasions chronicle the growth of Derek and Marion's children.

Due to Derek's life on the other side of the world, we did not interact as often as I would have liked. However, our early interactions left a special imprint on me, including his humor and the unique timbre of his voice. What sticks in my mind are Derek's stories of his unconventional friends, his love and penchant for cycling, his opinions about life in France, and the prospects for science in Australia. Sadly, now I only hear Derek in my mind; I miss his vibrant presence dearly.

Arthur Jaffe

Palle Jorgensen

Over his life, Derek led multiple collaboration teams. One research focus of collaboration inspired by Derek over multiple decades starting in the eighties, was the theory of continuous one-parameter groups and semigroups on Banach spaces, leading in turn to new noncommutative C^*-algebraic structures.
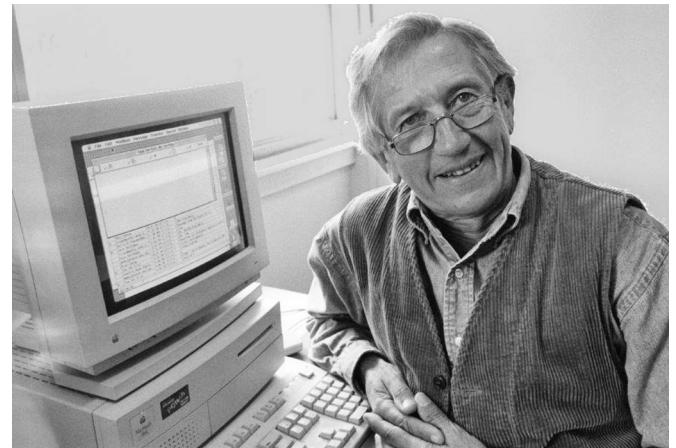
Figure 13. Derek Robinson at the computer, 1995.

Palle Jorgensen is a professor of mathematics at the University of Iowa. His email address is `palle-jorgensen@uiowa.edu`.

The collaborations started with annual research visits by Bratteli and me to Derek in Australia, during our winters. A bonus with this arrangement for us was that it allowed us to replace the cold winter in the North, with warm December–January Australia weather. Subsequently, the collaboration team expanded adding Charles Batty, Aki Kishimoto, and Dai Evans, among others. Key themes included an analysis of operator commutation relations, and their role in a unification of diverse areas in mathematics, in elementary particle physics, quantum-mechanical commutation relations, integration of Lie algebras, and unitary representations of noncompact Lie groups. A loss of a pioneer in mathematics always leaves us with a void. In the case of Derek, for many of us, it was also a loss of a dear friend.

Palle Jorgensen

Aki Kishimoto

While I was a PhD student under H. Araki, Derek Robinson was the first specialist who gave me encouragement via a letter (undoubtedly responding to my mentor's request). This was cemented by my yearlong visit to Marseille in October, 1977, a year later than originally planned. This was while he was preparing to leave for Sydney the following year. Although my interest turned out to be confined to a narrow realm of operator algebras, mostly anchored to the book which he was then writing with Ola Bratteli [2], his interest drifted away as time went by. But we managed to keep contact and occasionally meet until I received his last emails explaining his terminal health condition, adding that the paper he was revising then would be his last.

When I visited him in Sydney and later in Canberra, his coauthor Ola was almost always present. Even after he found collaborators in other fields, we talked about operator algebras related to mathematical physics in a broad sense. So he kept an interest in operator algebras from a physicist's point of view and shared his ideas with us. On

Aki Kishimoto is a professor emeritus in the mathematics department at Hokkaido University, Sapporo, Japan. His email address is akiksmt@r3.ucom .ne.jp.

Figure 14. Derek with best man, Tini Veltman, at his wedding.

my last visit to Canberra in 2006, we wrote a paper on flows. Although I met him a few more times after this collaboration, in Sapporo and Oslo, this was our last joint paper. What I remember well during this period is his kind response to the messages (which must have been somewhat distraught) I sent to a few friends after the 2011 earthquake and Fukushima disaster in Japan. He offered his guest house as a refuge from the nuclear fallout.

I suppose there have been many mathematical improvements to the results in their book after it was published. But now they do not seem to be that important. In hindsight, I was extremely fortunate to be able to witness Derek and Ola finishing writing their book and join their discussions on a couple of related problems.

Aki Kishimoto

Elliott Lieb

It is half a lifetime since Derek and I worked together, or even saw each other, but I remember our discussions and friendship with clarity. Perhaps the last time we met was when my wife, Christiane, and I enjoyed a stay at Derek's and Marion's house near Marseille on our way to a summer school in Cargèse. In any case, I cannot forget our work in Cambridge, Massachussetts, leading up to our 1972 paper [10] containing the bound on the speed of propagation of information in quantum spin systems. The paper took over three decades to surface, thanks to Matt Hastings, who cited it in 2004. This is remarkable since it is about twice as long as it takes the cicadas we have just experienced in New Jersey to hatch. It was fun working on it with him and I learned a lot in the process. I am grateful for that.

Figure 15. Derek with Elliott Lieb in Canberra, 1982.

There are other ways in which the mathematical physics community and I are indebted to Derek. Among them there is the fantastic two-volume treatise with Ola Bratteli [2], which established the foundations of our field. Equally important was Derek's foundational 1968 paper with Oscar Lanford on entropy [8], and its conjecture of quantum strong subadditivity, which I learned about from David Ruelle. It was a bold conjecture because, as it turns out, many of the simple properties of classical entropy fail in the quantum domain, yet this very complicated property actually carries through from classical to quantum.

The conjecture kept me busy for several years, up to my 1973 paper [9], which contains the concavity of $A \rightarrow$ trace $e^{K+\log A}$. This, together with Beth Ruskai's insight on

Elliott Lieb was a professor at the University of Sierra Leone, Yeshiva University, Northeastern University, and M.I.T. and is currently a professor emeritus at Princeton University. His email address is lieb@math.princeton.edu.

the importance of the joint concavity of conditional entropy, $S(A, B) - S(A)$, formed the basis of our joint work [11], which proved the fundamental Lanford–Robinson conjecture about entropy.

Those were glorious days in mathematical physics, generally, and Derek was one of its shining lights.
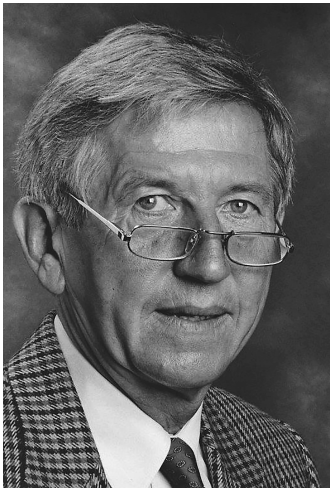

Elliott Lieb

Bruno Nachtergaele

To prepare for my official start as a graduate student in Leuven, André Verbeure assigned me some "light summer reading" from the two-volume book by Bratteli and Robinson. I found the material fascinating and inspiring (even if a bit dense!). The book has been my constant companion throughout graduate school and until this day. It also was while I was a graduate student in Leuven that Mark Fannes pointed out to me the finite group velocity theorem by Lieb and Robinson. I talked a lot to Mark and he frequently alerted me to beautiful results in mathematical physics that seemingly randomly popped up in our discussions. So, I was well aware of Derek Robinson's work but I cannot recall I ever met Derek in person. My only extended email exchange with him occurred in 2019 when he was reading the review paper I wrote with Sims and Young[13]. He set us straight about some of the original history and pointed us to a follow-up paper of his that we had been unaware of [16]. As my tribute to Derek's legacy, I will now discuss the subject of Lieb–Robinson bounds.

In nonrelativistic quantum mechanics of interacting degrees of freedom, correlations between all of them are generated in any amount of time. At first glance, it appears that signals can be transmitted instantaneously or at least at arbitrarily high speed. In a 1972 paper [10], Elliott Lieb and Derek Robinson showed that the dynamics of quantum spins on a lattice with finite-range interactions does, in fact, exhibit a bounded speed of propagation up to an exponentially small correction.

Bruno Nachtergaele is a Distinguished Professor of Mathematics at University of California, Davis. His email address is bxn@math.ucdavis.edu.

This result received some attention right away and it was recognized early on as a deep and important property of quantum many-body dynamics. It implied the existence of an approximate light cone which makes the non-relativistic dynamics look similar to relativistic quantum field theory of which the locality structure is a fundamental axiom. In spite of this, a series of important applications did not start to appear until more than three decades later. The citation history of Lieb and Robinson's paper, as shown by Google Scholar, hints at an event around 2004 that ignited a steadily increasing interest. Up to that point, the paper was cited only a couple dozen times (so, on average, less than once per year). Google Scholar now reports almost 1500 citations, and that figure continues to grow at a steady rate of about 150 per year.

Figure 16. Derek Robinson, circa 1990.

Matthew Hastings published several papers in 2004 that cite the Lieb–Robinson paper, most notably [5] and [6]. In the latter he gave a multi-dimensional version of the celebrated Lieb–Schultz–Mattis Theorem (LSM). In a eureka moment (which according to his recollection occurred while walking the Paris boulevards) he came to the realization that a finite propagation speed must hold for lattice systems with short range interactions and it was exactly what was required to complete his arguments. This was before becoming aware of the article by Lieb and Robinson with a proof of the property he needed. Another ingredient of Hastings's Lieb–Schultz–Mattis result is the exponential clustering theorem for quantum lattice systems. Several proofs of such a theorem were given in the context of axiomatic quantum field theory decades before, but it remained unproven folklore in statistical mechanics. The proof of exponential clustering in theories with a massive vacuum state or in the presence of a spectral gap above the ground state uses locality in an essential way. In quantum field theory this locality stems from the finite speed of light (Lorentz invariance). In statistical mechanics, a proof had to wait for Hastings's discovery of how to use the finite propagation speed provided by Lieb–Robinson bounds as a replacement for the finite speed of light [7, 12].

The exponential clustering theorem and the multi-dimensional Lieb–Schultz–Mattis theorem were the beginning of a sustained stream of applications. Lieb–Robinson bounds quickly developed into a fundamental component in quantum many-body theory and quantum information. Lieb–Robinson bounds are now established as a conceptual and practical tool in quantum theory. The vigorous pursuit of generalizations and improvements tailored to specific situations continues unabated.

Bruno Nachtergaele

Heide Narnhofer

Presumably Derek Robinson did not know about his influence on my scientific life, but it is a fact and I am happy that I can now express my gratitude.

I first met him when I was a student and he gave a talk in Vienna about how the thermodynamic quantities energy, entropy and pressure can be obtained in the language of "local quantum theory." This theory was fairly new at that time and Derek was one of its pioneers. His clarity about how physical quantities can be expressed as mathematical objects, and how on this basis rigorous considerations can lead to new expressions and realizable facts impressed me very much. I felt immediately that this was the language in which I could approach physical problems.

Three years later I was a postdoc at IHES in Bures and participated in a workshop of several researchers in mathematical physics, which included Derek. No talks were given, instead someone got up, starting with: "Recently I have thought about this problem…" and then the discussions and suggestions started. Derek spoke about the influence of boundary conditions, offered partial results and also the effect of not controlling the independence on Dirichlet and Neumann boundary conditions in the thermodynamic limit. R.B. Griffiths reacted, and I could see that out of intuition, confidence in intuition, experience in mathematical modeling, and the exchange of ideas, new ideas evolve and results emerge. In my subsequent research, I always tried to copy this combination of strong

Heide Narnhofer is a retired professor of mathematical physics, Faculty for Physics, University of Vienna. Her email address is `heide.narnhofer @univie.ac.at`.

belief mixed with criticism without being discouraged by it.

Figure 17. Derek as a young cyclist in the 1930s.

My last experience with Derek was in Marseille, when I asked him why pure thermodynamical states have more physical relevance than their mixtures. Derek immediately offered the explanation that it was due to their stability under the perturbation of their dynamics. There only remained the task of polishing the argument. Years later, I gave another explanation based on the idea of collapse in the framework of quantum history.

Derek turned more and more to mathematics and especially to the beauty of algebraic theory and its connection to differential theory, and he searched for results in this area. The theory of collapse that is based on randomness where probability theory is needed to find some structure was less attractive for him. I am sorry that we never had the possibility to discuss which approach reflects the reality better. This would have been very fruitful, because Derek was very open minded, and it was easy to communicate with him. He never gave the impression that a question was stupid, but instead was always encouraging and stimulating.

Heide Narnhofer

David Ruelle

A conference was organized in Derek's honor shortly before his death. At the time, David Ruelle sent the following letter to Derek, which we include here with Ruelle's permission. As you pointed out to me, our joint paper is still quoted after all

David Ruelle is a professor emeritus at Institut des Hautes Études Scientific (IHES). His email address is ruelle@ihes.fr.

these years. Of course science goes on, but our work took place at a crucial time and has left its imprint.

We have met over the years at various places, notably Bandol with Daniel Kastler when you seemed permanently established in France, and later Canberra where you invited me to visit the Australian National University. But I think that the first time we met was at the ETH in Zurich. This is when, after the death of Wolfgang Pauli, Res Jost congregated a little group of visitors at the "Theoretische Physik" first at Gloriastasse 35 then at Hochstrasse 60.

Let me remind you particularly of your Zurich period. Apart from you and me, young people who were around were Othmar Steinmann, Gianfausto Dell'Antonio, Klaus Hepp, Peter Curtius, Huzihiro Araki, Walter Hunziker, and others. We went for lunch at the Vogelsang where my wife Janine joined us. We could drink Apfelsaft there or alkoholfreies Rum Punch, and discuss forever on the future of physics. I also remember a party in an anti-atomic shelter of the Swiss army by the lake of Zurich. Those were the good days…

David Ruelle

Mary Beth Ruskai

In the 1960s, mathematical physicists seeking a firm foundation for statistical mechanics found that the framework of operator algebras was well-suited to this task. When I was a young postdoc in Geneva in 1970–1971, two of the leaders in this area, David Ruelle and Derek Robinson, gave troisième cycle courses in Lausanne with Derek's lectures focussed on thermodynamic pressure.

Earlier, Derek coauthored a pair of important papers on entropy. The first, with David Ruelle, studied classical systems, while the second, with Oscar Lanford, studied quantum systems. David suggested that I extend the results in the latter to von Neumann algebras with a well-behaved trace. Derek, then at the University Marseille, invited me to come for a week to coincide with a visit by Oscar. He arranged for me to stay at a small hotel near his home in the lovely town of Bandol on the Mediterranean. I fondly

Mary Beth Ruskai is retired. Her email address is mbruskai@gmail.com.

Figure 18. Derek at Berkeley around 1975.

remember meeting each afternoon at one of the cafes along the beach to discuss mathematics.

While doing this work, I realized that if the conditional entropy $S(\rho_{12}) - S(\rho_1)$ were concave it would imply a conjecture at the end of the paper by Lanford and Robinson. It may seem strange to suggest that the difference of two concave functions would be either concave or convex. However, Elliott Lieb proved that the conditional entropy is, indeed, concave and this was an important ingredient in my work with him proving the strong subadditivity of quantum entropy.

After Derek left Marseille we didn't cross paths again until 1997 when we met at the International Congress of Mathematical Physics in Brisbane. He was pleased to hear that the work on quantum entropy he began with Lanford in 1968 was now having an impact in quantum information theory.

Mary Beth Ruskai

Adam Sikora

The first time I met Derek was long ago, in October 1995. We met in Canberra, on the Australian National University campus, in a building that no longer exists. I came to Canberra on my first postdoc position, funded by Derek's ARC grant. It was easy to start working with him. Derek's research expertise and interest in mathematics were always

Adam Sikora is an associate professor of mathematics at Macarise University. His email address is adam.sikora@mq.edu.au.

extensive. I did not have to learn much of anything new—we just started to talk in order to find a topic appealing to both of us. And it was always a pleasure to meet and chat with him. He had an excellent sense of humor and lots of exciting things to share.

In our first joint work, we studied the Riesz transform in the setting of the Lie group. And it was the beginning of a quarter of a century of collaboration and friendship. After Lie groups, we investigated operators with periodic coefficients. Next, we were interested in degenerate elliptic operators. There are still many topics that I would like to discuss with Derek today, but sadly this is not possible anymore.

Derek always wanted to learn new things. Even in his sixties or seventies, he was not afraid of starting a study in a new research area and learning new things. The evolution of his research expertise is impressive and broad: from physics to C*algebras, Lie groups, Riesz transform, operators with periodic coefficients, degenerate elliptic operators, and many more areas.

When it was clear that Derek was terminally ill and his prognosis was not good, I had a chance to talk with him a few times over the phone. Derek told me that he had submitted a new paper to the *Journal of Functional Analysis*, that he was frail and struggled to revise the manuscripts, and how happy he was when it was accepted. It made me realize how significant mathematics was for him. He was very tired and not able to chat for too long, but we were able to discuss many other things. I am surprised by how important it was for me to talk with him and to thank him for our collaboration and all the other things he did for me.

Derek had the ability and wisdom to cherish every day of his life—how to use any moment and enjoy it. As we all know, he loved cycling, but when cycling became too risky for his age, he took up race-walking to appreciate good weather and stay fit and healthy. Derek loved to be active, which was always a part of his personality. Above all, he always wanted to do some new mathematics, make some exciting calculations, write and explain them to others, and discuss them over coffee with friends. Derek used and enjoyed every day of his life as much as possible.

Adam Sikora

Barry Simon

I first met Derek in early in 1973. Freshly tenured at Princeton, I spent the academic year 1972–1973 visiting three great centers of European mathematical physics: the fall at IHES near Paris, the spring at ETH in Zurich, and the winter at what was jokingly called the free university of Bandol. This was the name given to the remarkable group that Daniel Kastler had put together at the Universities of Aix-Marseille, Toulon, and mainly at the CNRS in Marseille where I was officially visiting. Two of the other senior members of the group were Derek and Alex Grossman. Daniel and Derek lived in the village of Bandol about 50 km from CNRS and Alex lived in the quaint fishing village of Cassis half way between, which is where my wife and I stayed. Much research was done informally in the villages, hence the name.

Derek was lively and outspoken. I was somewhat shocked by his references to *damned frogs* although I eventually learned about his travails with French bureaucracy over arranging foreign visitors and building his house and understood this usage better. In any event, Derek did it with such charm that it seemed all right in any event.

My second and more intense interaction with Derek was when he invited me to visit in the summer of 1983, or perhaps I should say the winter since the visit was to Canberra where he'd relocated. The visit almost didn't happen. My youngest son was born in Dec. 1982 and by the time we were able to get a birth certificate and use it to get a passport for him, it was the end of April before we sent the visa application to the Australian consulate. The visa officer was making unreasonable demands about a medical form so, in the era before email and with expensive international phone calls, I sent a telex to Derek explaining that I might not be able to come. He called and told me to hang tight. Two days later, the formerly officious visa officer called: "Sir, I am anxious to issue your visa but I need you to return your passports." "What about the medical form?" "Oh you don't need that, sir." Derek had performed a miracle.

It was a very important visit for me scientifically. Michael Berry was visiting the physics department and told me about some recent work he'd done which led to my paper on what I called *Berry's phase* surely my most famous work in the physics community.

Brian Davies was also visiting and we worked hard on some results involving a new concept that needed a name. We found a notion which was stronger than the notions of *hypercontractive semigroups* and *supercontractive semigroups*. I

Barry Simon is the IBM Professor of Mathematics and Theoretical Physics, Emeritus at Caltech. His email address is bsimon@caltech.edu.


Figure 19. Derek with Aubrey Truman at Simonfest, 2006.

remember the meeting we had in Derek's office trying to figure out a term stronger than "hyper" and "'super." After we joked about "super-duper," Derek suggested "ultra" and so was born *ultracontractive semigroups*, a term with almost 10,000 hits on Google!

While our research interests weren't distant, they weren't really close except for two lovely things that Derek had in his book on the Thermodynamic Pressure [15] which involved my focal interest of Schrödinger operators. In our work on Thomas Fermi theory, Elliott Lieb and I used a variant of a technique of Weyl which André Martin had used to get the large coupling limit of the number of bound states of a Schrödinger operator but we learned Derek had it independently. And Derek had proven a conjecture of Kato on monotone limits of forms earlier than I had.

In recent years, Derek and I frequently swapped emails—exchanges that were illuminating and often fun. I miss him.
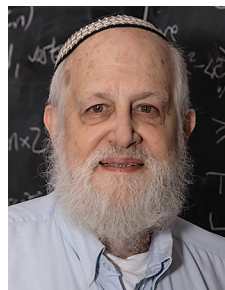
Barry Simon

A.F.M. ter Elst

Almost unnoticed Derek Robinson fulfilled major administrative responsibilities. For example, he was the Chairman of the Board of the Institute of Advanced Studies at the ANU from 1988 till 1992. In order to stay research active, in 1988 he decided to write the book [17]. On my arrival day as a postdoc in Canberra in mid 1990, he gave me the first two chapters of the manuscript with the message that it was the final draft and that I certainly would find a topic to work on. I was deeply impressed by how well it was written. It contained a wealth of information and many things to work on. The "final draft" still got two revisions.

Figure 20. Derek in Canberra, 1982.

The book covered (possibly higher-order and complex) strongly elliptic operators and sublaplacians on Lie groups. Derek used a Nash inequality in order to obtain Gaussian bounds for the kernel of the semigroup associated with elliptic operators, extending Langland's PhD results. Together we managed to extend many theorems to the setting of possibly higher-order complex subelliptic operators on a Lie group, even allowing for different weights in different directions, as happens for positive Rockland operators on graded Lie groups. We next describe a striking example. Let L be the left regular representation of $SO(3)$ in $L_2(SO(3))$. Further let A_k be the infinitesimal generator of the natural one-parameter groups. Then the ordinary Laplacian is the operator $A_1^2 + A_2^2 + A_3^2$ and a sublaplacian is $A_1^2 + A_2^2$. Derek investigated these operators. Now take as an example $H = A_1^4 - A_2^6$ or $H = A_1^4 - A_2^6 + A_1^2 A_2^3$. Later,

A.F.M. ter Elst is a professor of mathematics at the University of Auckland. His email address is terelst@math.auckland.ac.nz.

Derek and I proved that operators like H are also the (minus) generator of a holomorphic C_0-semigroup which has a kernel satisfying Gaussian type bounds, including all its higher-order derivatives. This even provided a characterisation of these weighted subcoercive operators.

A different challenge is to obtain asymptotics for the semigroup and its kernel for large time. For operators with periodic coefficients a major step was obtained with techniques of homogenization theory. The details were written in another book [4]. After that Derek got more and more interested in degenerate elliptic operators and the relation with Hardy inequalities [19].

A. F. M. ter Elst

References

[1] Ola Bratteli and Derek W. Robinson, *Unbounded derivations of C*-algebras*, Comm. Math. Phys. **42** (1975), 253–268. MR377526

[2] Ola Bratteli and Derek W. Robinson, *Operator algebras and quantum-statistical mechanics. II: Equilibrium states. Models in quantum-statistical mechanics*, Texts and Monographs in Physics, Springer-Verlag, New York-Berlin, 1981. MR611508

[3] Sergio Doplicher, Daniel Kastler, and Derek W. Robinson, *Covariance algebras in field theory and statistical mechanics*, Comm. Math. Phys. **3** (1966), 1–28. MR205095

[4] Nick Dungey, A. F. M. ter Elst, and Derek W. Robinson, *Analysis on Lie groups with polynomial growth*, Progress in Mathematics, vol. 214, Birkhäuser Boston, Inc., Boston, MA, 2003, DOI 10.1007/978-1-4612-2062-6. MR2000440

[5] M. B. Hastings, *Locality in Quantum and Markov Dynamics on Lattices and Networks*, Phys. Rev. Lett. **93** (2004), 140402.

[6] M. B. Hastings, *Lieb-Schultz-Mattis in Higher Dimensions*, Phys. Rev. **B69** (2004), 104431.

[7] Matthew B. Hastings and Tohru Koma, *Spectral gap and exponential decay of correlations*, Comm. Math. Phys. **265** (2006), no. 3, 781–804, DOI 10.1007/s00220-006-0030-4. MR2231689

[8] O. E. Lanford, III and D. W. Robinson, *Mean entropy of states in quantum statistical mechanics*, J. Math. Phys. **9** (1968), 1120–1125.

[9] Elliott H. Lieb, *Convex trace functions and the Wigner-Yanase-Dyson conjecture*, Advances in Math. **11** (1973), 267–288, DOI 10.1016/0001-8708(73)90011-X. MR332080

[10] Elliott H. Lieb and Derek W. Robinson, *The finite group velocity of quantum spin systems*, Comm. Math. Phys. **28** (1972), 251–257. MR312860

[11] Elliott H. Lieb and Mary Beth Ruskai, *Proof of the strong subadditivity of quantum-mechanical entropy*, J. Mathematical Phys. **14** (1973), 1938–1941, DOI 10.1063/1.1666274. With an appendix by B. Simon. MR345558

[12] Bruno Nachtergaele and Robert Sims, *Lieb-Robinson bounds and the exponential clustering theorem*, Comm. Math. Phys. **265** (2006), no. 1, 119–130, DOI 10.1007/s00220-006-1556-1. MR2217299

[13] Bruno Nachtergaele, Robert Sims, and Amanda Young, *Lieb-Robinson bounds, the spectral flow, and stability of the spectral gap for lattice fermion systems*, Mathematical problems in quantum physics, Contemp. Math., vol. 717, Amer. Math. Soc., [Providence], RI, [2018] ©2018, pp. 93–115, DOI 10.1090/conm/717/14443. MR3869135

[14] Derek W. Robinson, *Statistical mechanics of quantum spin systems. II*, Comm. Math. Phys. **7** (1968), 337–348. MR228246

[15] Derek W. Robinson, *The thermodynamic pressure in quantum statistical mechanics*, Lecture Notes in Physics, Vol. 9, Springer-Verlag, Berlin-New York, 1971. MR0432122

[16] D. W. Robinson, *Properties of propagation of quantum spin systems*, J. Australian Math. Soc. **19** (Series B), 387–399 (1976).

[17] Derek W. Robinson, *Elliptic operators and Lie groups*, Oxford Mathematical Monographs, The Clarendon Press, Oxford University Press, New York, 1991. Oxford Science Publications. MR1144020

[18] D. W. Robinson, *Life with Ola*, part of *Ola Bratteli and His Diagrams*, Notices Amer. Math. Soc. **67** (2020), 665–675

[19] Derek W. Robinson, *The weighted Hardy inequality and self-adjointness of symmetric diffusion operators*, J. Funct. Anal. **281** (2021), no. 4, Paper No. 109067, 34, DOI 10.1016/j.jfa.2021.109067. MR4249773

[20] Derek W. Robinson and David Ruelle, *Mean entropy of states in classical statistical mechanics*, Comm. Math. Phys. **5** (1967), 288–300. MR225553