Ed Nelson's Work in Analysis, Especially Related to Quantum Field Theory

Barry Simon

IBM Professor of Mathematics and Theoretical Physics
California Institute of Technology
Pasadena, CA, U.S.A.

Lunch Talk at Princeton, April 22, 2015

Two Great Contributions

This talk will focus on some of Ed Nelson's contributions to analysis. Ed also made significant contributions to probability and to mathematical logic. Moreover, other than one multi-part slide, I will focus on two subjects where Ed's seminal contributions have had enormous impact (and which were singled out in the citation for his 1995 Steele Prize for Seminal Contributions to Research):

Two Great Contributions

This talk will focus on some of Ed Nelson's contributions to analysis. Ed also made significant contributions to probability and to mathematical logic. Moreover, other than one multi-part slide, I will focus on two subjects where Ed's seminal contributions have had enormous impact (and which were singled out in the citation for his 1995 Steele Prize for Seminal Contributions to Research):

1. **Hypercontractive Semigroups** In 1966, Ed defined what came to be called hypercontractive semigroups.

Two Great Contributions

This talk will focus on some of Ed Nelson's contributions to analysis. Ed also made significant contributions to probability and to mathematical logic. Moreover, other than one multi-part slide, I will focus on two subjects where Ed's seminal contributions have had enormous impact (and which were singled out in the citation for his 1995 Steele Prize for Seminal Contributions to Research):

1. **Hypercontractive Semigroups** In 1966, Ed defined what came to be called hypercontractive semigroups.
2. **Euclidean (Constructive) Quantum Field Theory** In 1972-73, Ed was the initiator of what was a revolution in the mathematical theory of constructing relativistic quantum fields, so that after his work, essentially all research was done in a Euclidean framework.

Two Great Contributions

Hypercontractivity appeared initially as a tool in studying boundedness from below and self-adjointness but from the earlier 70's onwards it spread wings so that a Google search finds over 100,000 entries and an infinitesmal version ("log Sobolev inequalties") was used for example in Perlman's solution of the Poincaré conjecture.

Two Great Contributions

Hypercontractivity appeared initially as a tool in studying boundedness from below and self-adjointness but from the earlier 70's onwards it spread wings so that a Google search finds over 100,000 entries and an infinitesmal version ("log Sobolev inequalties") was used for example in Perlman's solution of the Poincaré conjecture.

Two common themes in both these works were the idea of path integrals for semigroups and an extensive probabilistic intuition.

Two Great Contributions

Hypercontractivity appeared initially as a tool in studying boundedness from below and self-adjointness but from the earlier 70's onwards it spread wings so that a Google search finds over 100,000 entries and an infinitesmal version ("log Sobolev inequalties") was used for example in Perlman's solution of the Poincaré conjecture.

Two common themes in both these works were the idea of path integrals for semigroups and an extensive probabilistic intuition. Another is that both were incredibly innovative – developing strikingly new approaches which turned out to be incredibly powerful and useful.

Other Work in Analysis

I should at least mention some of Ed's other contributions to analysis:

Other Work in Analysis

I should at least mention some of Ed's other contributions to analysis:

- His proof of the Feynman-Kac formula using the Trotter product formula

Other Work in Analysis

I should at least mention some of Ed's other contributions to analysis:

- His proof of the Feynman-Kac formula using the Trotter product formula
- Nelson's model - a non-relativistic quantum field model with zero mass particles that is useful in the study of the infrared problem and remains heavily studied (e.g. 20 papers since 2010)

Other Work in Analysis

I should at least mention some of Ed's other contributions to analysis:

- His proof of the Feynman-Kac formula using the Trotter product formula
- Nelson's model - a non-relativistic quantum field model with zero mass particles that is useful in the study of the infrared problem and remains heavily studied (e.g. 20 papers since 2010)
- His theory of analytic vectors providing both a self-adjointness technique and a simple way to relate Lie algebras and Lie group representations

Other Work in Analysis

I should at least mention some of Ed's other contributions to analysis:

- His proof of the Feynman-Kac formula using the Trotter product formula
- Nelson's model - a non-relativistic quantum field model with zero mass particles that is useful in the study of the infrared problem and remains heavily studied (e.g. 20 papers since 2010)
- His theory of analytic vectors providing both a self-adjointness technique and a simple way to relate Lie algebras and Lie group representations
- His example of a quantum problem which has a unique self-adjoint extension even though the classical analog gets to infinity in finite time (unpublished, but it appeared in Reed-Simon, Vol 2 with attribution).

Other Work in Analysis

- His elegant proof of Loiuville's theorem for positive harmonic functions from the mean value theorem. It appeared in a *Proc. AMS* paper of 9 lines (*lines, not pages*) without a single formula or the words "definition" or "theorem".

Other Work in Analysis

- His elegant proof of Loiuville's theorem for positive harmonic functions from the mean value theorem. It appeared in a *Proc. AMS* paper of 9 lines (*lines, not pages*) without a single formula or the words "definition" or "theorem".
- His self-adjointness result for non-semibounded operators using commutators (improving on an earlier result of Glimm-Jaffe) with a lovely application by Faris-Lavine to Stark Hamiltonians (more later)

Other Work in Analysis

- His elegant proof of Loiuville's theorem for positive harmonic functions from the mean value theorem. It appeared in a *Proc. AMS* paper of 9 lines (*lines, not pages*) without a single formula or the words "definition" or "theorem".
- His self-adjointness result for non-semibounded operators using commutators (improving on an earlier result of Glimm-Jaffe) with a lovely application by Faris-Lavine to Stark Hamiltonians (more later)
- His proof of diamagnetic inequalities which some have called *Nelson-Simon inequalties*

Other Work in Analysis

- His elegant proof of Loiuville's theorem for positive harmonic functions from the mean value theorem. It appeared in a *Proc. AMS* paper of 9 lines (*lines, not pages*) without a single formula or the words "definition" or "theorem".
- His self-adjointness result for non-semibounded operators using commutators (improving on an earlier result of Glimm-Jaffe) with a lovely application by Faris-Lavine to Stark Hamiltonians (more later)
- His proof of diamagnetic inequalities which some have called *Nelson-Simon inequalties*
- Some influential Lecture Notes including on quadratic form methods

Other Work in Analysis

- His elegant proof of Loiuville's theorem for positive harmonic functions from the mean value theorem. It appeared in a *Proc. AMS* paper of 9 lines (*lines, not pages*) without a single formula or the words "definition" or "theorem".
- His self-adjointness result for non-semibounded operators using commutators (improving on an earlier result of Glimm-Jaffe) with a lovely application by Faris-Lavine to Stark Hamiltonians (more later)
- His proof of diamagnetic inequalities which some have called *Nelson-Simon inequalties*
- Some influential Lecture Notes including on quadratic form methods
- Results in non-standard analysis and in stochastic mechanics

Hamiltonian CQFT

A major accomplishment of mathematical physicists in the period from 1965-1985 was the construction of interacting quantum field theories in 2 and 3 space time dimensions but not, alas, 4 dimensions.

Hamiltonian CQFT

A major accomplishment of mathematical physicists in the period from 1965-1985 was the construction of interacting quantum field theories in 2 and 3 space time dimensions but not, alas, 4 dimensions. When Ed started, there was essentially no prior work. I want to describe the problem he faced and solved that led to the start of the theory of hypercontractive semigroups.

Hamiltonian CQFT

A major accomplishment of mathematical physicists in the period from 1965-1985 was the construction of interacting quantum field theories in 2 and 3 space time dimensions but not, alas, 4 dimensions. When Ed started, there was essentially no prior work. I want to describe the problem he faced and solved that led to the start of the theory of hypercontractive semigroups.

The idea was to generalize the anharmonic oscillator, $H_0 + x^4$ where H_0 is the Hamiltonian operator of a harmonic oscillator. One replaces the harmonic oscillator by the Hamiltonain, H_0 of a massive free field and formally wants to look at

$$H = H_0 + \int \phi^4(x)\,dx$$

Hamiltonian CQFT

The first issue is that the integral is over all of $\mathbb{R}$ and so will surely diverge applied to any state. One puts in some kind of spatial cutoff which eventually one wants remove.

Hamiltonian CQFT

The first issue is that the integral is over all of $\mathbb{R}$ and so will surely diverge applied to any state. One puts in some kind of spatial cutoff which eventually one wants remove. In fact, Ed put things in a periodic box so the free field is described by a family of infinitely many decoupled simple harmonic oscillators describing excitations of the field by periodic plane waves.

Hamiltonian CQFT

The first issue is that the integral is over all of $\mathbb{R}$ and so will surely diverge applied to any state. One puts in some kind of spatial cutoff which eventually one wants remove. In fact, Ed put things in a periodic box so the free field is described by a family of infinitely many decoupled simple harmonic oscillators describing excitations of the field by periodic plane waves.

But the real problem involves the fact that $\phi(x)$ isn't just an operator; it is an operator valued distribution. So taking powers is impossible. This is the simplest of case of the famous infinities that plague formal quantum field theory.

Hamiltonian CQFT

The solution in this case is easy - one cuts off the field (by considering only finitely many modes)

Hamiltonian CQFT

The solution in this case is easy - one cuts off the field (by considering only finitely many modes) and replaces $\phi^4(x)$ by $c_N^4 h_4(\phi_N(x)/c_N)$ where h_4 is the monic fourth Hermite polynomial and c_N is a constant that goes to infinity as $\log(N)$. One can then take a limit as $N \to \infty$.

Hamiltonian CQFT

The solution in this case is easy - one cuts off the field (by considering only finitely many modes) and replaces $\phi^4(x)$ by $c_N^4 h_4(\phi_N(x)/c_N)$ where h_4 is the monic fourth Hermite polynomial and c_N is a constant that goes to infinity as $\log(N)$. One can then take a limit as $N \to \infty$.

This "Wick ordered" $\phi^4(x)$ is a well defined distribution and one can form the integral over the circle and get a well defined operator, V. But since the Hermite polynomial is negative at some values, and is being multiplied by an infinite constant, V is no longer bounded from below.

Hamiltonian CQFT

The solution in this case is easy - one cuts off the field (by considering only finitely many modes) and replaces $\phi^4(x)$ by $c_N^4 h_4(\phi_N(x)/c_N)$ where h_4 is the monic fourth Hermite polynomial and c_N is a constant that goes to infinity as $\log(N)$. One can then take a limit as $N \to \infty$.

This "Wick ordered" $\phi^4(x)$ is a well defined distribution and one can form the integral over the circle and get a well defined operator, V. But since the Hermite polynomial is negative at some values, and is being multiplied by an infinite constant, V is no longer bounded from below. The issue that Ed faced and conquered was proving that $H_0 + V$ is bounded from below.

Hamiltonian CQFT

The solution in this case is easy - one cuts off the field (by considering only finitely many modes) and replaces $\phi^4(x)$ by $c_N^4 h_4(\phi_N(x)/c_N)$ where h_4 is the monic fourth Hermite polynomial and c_N is a constant that goes to infinity as $\log(N)$. One can then take a limit as $N \to \infty$.

This "Wick ordered" $\phi^4(x)$ is a well defined distribution and one can form the integral over the circle and get a well defined operator, V. But since the Hermite polynomial is negative at some values, and is being multiplied by an infinite constant, V is no longer bounded from below. The issue that Ed faced and conquered was proving that $H_0 + V$ is bounded from below. The first thing Ed realized is that to deal with the infinite number of modes, it was natural to take an infinite product space and to do analysis on such a space one wants infinite products of probability measures.

Hamiltonian CQFT

For each mode, it is natural to take the x-space quantum probability density in the its ground state which gives an Gaussian measure on $\mathbb{R}^\infty$. Ed's paper had three critical steps:

Hamiltonian CQFT

For each mode, it is natural to take the x-space quantum probability density in the its ground state which gives an Gaussian measure on $\mathbb{R}^\infty$. Ed's paper had three critical steps:

1. L^p **properties of** e^{-V}. In this realization, V is a multiplication operator by a function not bounded below. Ed realized because the divergence is only logarithmic, e^{-V} lies in every L^p space, $p < \infty$.

Hamiltonian CQFT

For each mode, it is natural to take the x-space quantum probability density in the its ground state which gives an Gaussian measure on $\mathbb{R}^\infty$. Ed's paper had three critical steps:

1. L^p **properties of** e^{-V}. In this realization, V is a multiplication operator by a function not bounded below. Ed realized because the divergence is only logarithmic, e^{-V} lies in every L^p space, $p < \infty$.
2. **Hypercontractive semigroups**. He proved some L^p smoothing of e^{-tH_0} that we now call hypercontractivity (of which more below)

Hamiltonian CQFT

For each mode, it is natural to take the x-space quantum probability density in the its ground state which gives an Gaussian measure on $\mathbb{R}^\infty$. Ed's paper had three critical steps:

1. L^p **properties of** e^{-V}. In this realization, V is a multiplication operator by a function not bounded below. Ed realized because the divergence is only logarithmic, e^{-V} lies in every L^p space, $p < \infty$.
2. **Hypercontractive semigroups**. He proved some L^p smoothing of e^{-tH_0} that we now call hypercontractivity (of which more below)
3. **Boundedness from below**. He proved that if V is a multiplication operator with e^{-V} in every L^p space, $p < \infty$ and if H_0 generates a hypercontractive semigroup, then $H_0 + V$ is bounded below.

Definition of Hypercontractivity

An L^p **contractive semigroup** is a self-adjoint semigroup, e^{-tA} on $L^2(X, d\mu)$ that is a contraction on each $L^p(X, d\mu)$.

Definition of Hypercontractivity

An L^p **contractive semigroup** is a self-adjoint semigroup, e^{-tA} on $L^2(X, d\mu)$ that is a contraction on each $L^p(X, d\mu)$. A common way they arise is if the semigroup takes positve functions to positive functions and $\mathbb{1}$ to itself (because then $|e^{-tA}f(x)| \leq \|f\|_\infty e^{-tA}\mathbb{1}(x)$).

Definition of Hypercontractivity

An L^p **contractive semigroup** is a self-adjoint semigroup, e^{-tA} on $L^2(X, d\mu)$ that is a contraction on each $L^p(X, d\mu)$. A common way they arise is if the semigroup takes positve functions to positive functions and $\mathbb{1}$ to itself (because then $|e^{-tA}f(x)| \leq \|f\|_\infty e^{-tA}\mathbb{1}(x)$).

A **hypercontractive semigroup** is an L^p contractive semigroup on a probability measure space so that for some T, e^{-TA} maps L^2 to L^4.

Definition of Hypercontractivity

An L^p **contractive semigroup** is a self-adjoint semigroup, e^{-tA} on $L^2(X, d\mu)$ that is a contraction on each $L^p(X, d\mu)$. A common way they arise is if the semigroup takes positve functions to positive functions and $\mathbb{1}$ to itself (because then $|e^{-tA}f(x)| \leq \|f\|_\infty e^{-tA}\mathbb{1}(x)$).

A **hypercontractive semigroup** is an L^p contractive semigroup on a probability measure space so that for some T, e^{-TA} maps L^2 to L^4. Ed didn't make a formal definition of this form but he did prove the L^2 to L^4 result for the periodic boundary condition free field.

Definition of Hypercontractivity

The formal definition was made in a 1972 paper I wrote with Høegh-Krohn where the name hypercontractive first appeared.

Definition of Hypercontractivity

The formal definition was made in a 1972 paper I wrote with Høegh-Krohn where the name hypercontractive first appeared. Shortly afterwards, Ed complained to me "You know – it shouldn't be hypercontractive because you only assume boundedness from L^2 to L^4. It should be hyperbounded."

Definition of Hypercontractivity

The formal definition was made in a 1972 paper I wrote with Høegh-Krohn where the name hypercontractive first appeared. Shortly afterwards, Ed complained to me "You know – it shouldn't be hypercontractive because you only assume boundedness from L^2 to L^4. It should be hyperbounded." I replied that hypercontractive sounded a lot better and, indeed the name took off and Ed used it in a number of papers.

Definition of Hypercontractivity

The formal definition was made in a 1972 paper I wrote with Høegh-Krohn where the name hypercontractive first appeared. Shortly afterwards, Ed complained to me "You know – it shouldn't be hypercontractive because you only assume boundedness from L^2 to L^4. It should be hyperbounded." I replied that hypercontractive sounded a lot better and, indeed the name took off and Ed used it in a number of papers. Still, he used it as a running gag - when I saw him 18 months ago, he smiled and said "You know – it shouldn't have been called hypercontractive, but hyperbounded".

Semiboundedness

While not his original proof of semiboundedness from hypercontractivity + L^p properties of e^{-tV}, here a general version going back to Segal:

Semiboundedness

While not his original proof of semiboundedness from hypercontractivity + L^p properties of e^{-tV}, here a general version going back to Segal:

Segal's Lemma For bounded operators A and B, we have that $\|e^{A+B}\| \leq \|e^{A/2}e^{B}e^{A/2}\|$ where the norms are operator norm.

Semiboundedness

While not his original proof of semiboundedness from hypercontractivity + L^p properties of e^{-tV}, here a general version going back to Segal:

Segal's Lemma For bounded operators A and B, we have that $\|e^{A+B}\| \leq \|e^{A/2}e^{B}e^{A/2}\|$ where the norms are operator norm.

By taking $A = -2TH_0$ and $B = -2TV$ and using that $e^{A/2}$ maps L^2 to L^4 and also $L^{4/3}$ to L^2 by duality and that e^B maps L^4 to $L^{4/3}$ (and taking limits of bounded cutoffs), one gets hypercontractive semiboundedness.

Semiboundedness

To prove Segal's Lemma apply the three line theorem to the analytic operator valued function $F(z) = e^{zB/2}e^{zA/2}$ on $\{z|0 \leq \Re(z) \leq 1\}$, to see that

Semiboundedness

To prove Segal's Lemma apply the three line theorem to the analytic operator valued function $F(z) = e^{zB/2}e^{zA/2}$ on $\{z|0 \leq \Re(z) \leq 1\}$, to see that

$$\|e^{B/2^n}e^{A/2^n}\| \leq \|e^{B/2}e^{A/2}\|^{2^{-(n-1)}}$$

.

Semiboundedness

To prove Segal's Lemma apply the three line theorem to the analytic operator valued function $F(z) = e^{zB/2}e^{zA/2}$ on $\{z|0 \leq \Re(z) \leq 1\}$, to see that

$$\|e^{B/2^n}e^{A/2^n}\| \leq \|e^{B/2}e^{A/2}\|^{2^{-(n-1)}}$$

.

Using $\|C^m\| \leq \|C\|^m$ and $\|D^*D\| = \|D\|^2$, we get

Semiboundedness

To prove Segal's Lemma apply the three line theorem to the analytic operator valued function $F(z) = e^{zB/2}e^{zA/2}$ on $\{z|0 \leq \Re(z) \leq 1\}$, to see that

$$\|e^{B/2^n}e^{A/2^n}\| \leq \|e^{B/2}e^{A/2}\|^{2^{-(n-1)}}$$

.

Using $\|C^m\| \leq \|C\|^m$ and $\|D^*D\| = \|D\|^2$, we get

$$\|(e^{B/2^n}e^{A/2^n})^{2^n}\| \leq \|e^{A/2}e^{B}e^{A/2}\|$$

.

Semiboundedness

To prove Segal's Lemma apply the three line theorem to the analytic operator valued function $F(z) = e^{zB/2}e^{zA/2}$ on $\{z|0 \leq \Re(z) \leq 1\}$, to see that

$$\|e^{B/2^n}e^{A/2^n}\| \leq \|e^{B/2}e^{A/2}\|^{2^{-(n-1)}}$$

.

Using $\|C^m\| \leq \|C\|^m$ and $\|D^*D\| = \|D\|^2$, we get

$$\|(e^{B/2^n}e^{A/2^n})^{2^n}\| \leq \|e^{A/2}e^{B}e^{A/2}\|$$

.

The Trotter product formula completes the proof.

Later devolvements: Some highpoints

Here are some of the major developments focusing on those close to Ed's original purpose and/or major ones

Later devolvements: Some highpoints

Here are some of the major developments focusing on those close to Ed's original purpose and/or major ones (ignores applications to probability and to computer science and the non-commutative (Fermion) version):

Later devolvements: Some highpoints

Here are some of the major developments focusing on those close to Ed's original purpose and/or major ones (ignores applications to probability and to computer science and the non-commutative (Fermion) version):

- Glimm and Federbush refine original semiboundedness results (1968-69)

Later devolvements: Some highpoints

Here are some of the major developments focusing on those close to Ed's original purpose and/or major ones (ignores applications to probability and to computer science and the non-commutative (Fermion) version):

- Glimm and Federbush refine original semiboundedness results (1968-69)
- Segal and Simon-Høegh-Krohn discuss self-adjointness and abstract versions (1970-71)

Later devolvements: Some highpoints

Here are some of the major developments focusing on those close to Ed's original purpose and/or major ones (ignores applications to probability and to computer science and the non-commutative (Fermion) version):

- Glimm and Federbush refine original semiboundedness results (1968-69)
- Segal and Simon-Høegh-Krohn discuss self-adjointness and abstract versions (1970-71)
- In his papers on EQFT, Nelson found optimal hypercontractive estimates in one and then more dimensions.

Later devolvements: Some highpoints

Here are some of the major developments focusing on those close to Ed's original purpose and/or major ones (ignores applications to probability and to computer science and the non-commutative (Fermion) version):

- Glimm and Federbush refine original semiboundedness results (1968-69)
- Segal and Simon-Høegh-Krohn discuss self-adjointness and abstract versions (1970-71)
- In his papers on EQFT, Nelson found optimal hypercontractive estimates in one and then more dimensions. A suitably normalized H_0 has $\exp(-tH_0)$ bounded from $L^p \to L^q$ if and only if $e^{-t} \leq \sqrt{p - 1/q - 1}$ and then it is a contraction (1971-72).

Later devolvements: Some highpoints

- In the most significant development after Ed's original paper, Gross found the infinitesimal version of hypercontractivity (called *log Sobolev inequalities*) and how to go back and forth (1972-73).

Later devolvements: Some highpoints

- In the most significant development after Ed's original paper, Gross found the infinitesimal version of hypercontractivity (called *log Sobolev inequalities*) and how to go back and forth (1972-73).
- Gross also had the idea of studying hypercontractivity for semigroups on $\mathbb{R}^2$ rather than an infinite dimensional L^2 and got Nelson's optimal result by getting the two dimensional best constants (now called Bonami-Gross inequalities) and using a central limit theorem to get a Gaussian measure result. (1972-73)

Later devolvements: Some highpoints

- In the most significant development after Ed's original paper, Gross found the infinitesimal version of hypercontractivity (called *log Sobolev inequalities*) and how to go back and forth (1972-73).
- Gross also had the idea of studying hypercontractivity for semigroups on $\mathbb{R}^2$ rather than an infinite dimensional L^2 and got Nelson's optimal result by getting the two dimensional best constants (now called Bonami-Gross inequalities) and using a central limit theorem to get a Gaussian measure result. (1972-73)
- Motivated by Gross, Beckner found optimal constants in the Hausdorff-Young inequalities. (1974)

Later devolvements: Some highpoints

- In the most significant development after Ed's original paper, Gross found the infinitesimal version of hypercontractivity (called *log Sobolev inequalities*) and how to go back and forth (1972-73).
- Gross also had the idea of studying hypercontractivity for semigroups on $\mathbb{R}^2$ rather than an infinite dimensional L^2 and got Nelson's optimal result by getting the two dimensional best constants (now called Bonami-Gross inequalities) and using a central limit theorem to get a Gaussian measure result. (1972-73)
- Motivated by Gross, Beckner found optimal constants in the Hausdorff-Young inequalities. (1974)
- Davies-Simon develop the theory of ultracontactivy (instantaneous map from $L^2 \to L^\infty$). (1984)

Later devolvements: Some highpoints

- In the most significant development after Ed's original paper, Gross found the infinitesimal version of hypercontractivity (called *log Sobolev inequalities*) and how to go back and forth (1972-73).
- Gross also had the idea of studying hypercontractivity for semigroups on $\mathbb{R}^2$ rather than an infinite dimensional L^2 and got Nelson's optimal result by getting the two dimensional best constants (now called Bonami-Gross inequalities) and using a central limit theorem to get a Gaussian measure result. (1972-73)
- Motivated by Gross, Beckner found optimal constants in the Hausdorff-Young inequalities. (1974)
- Davies-Simon develop the theory of ultracontactivy (instantaneous map from $L^2 \to L^\infty$). (1984)
- Perelman uses log Sobolev inequalities in his work on Ricci flow that proved the Poincaré conjecture. (2008)

Gaussian processes

To explain Ed's contribution to Euclidean Quantum Field Theory (EQFT), it is useful to first look at path integrals for ordinary quantum mechanics.

Gaussian processes

To explain Ed's contribution to Euclidean Quantum Field Theory (EQFT), it is useful to first look at path integrals for ordinary quantum mechanics. A finite number of jointly Gaussian random variables have a probability distribution determined by their covariance $\mathbb{E}(X_i X_j)$ and any positive definite matrix can occur.

Gaussian processes

To explain Ed's contribution to Euclidean Quantum Field Theory (EQFT), it is useful to first look at path integrals for ordinary quantum mechanics. A finite number of jointly Gaussian random variables have a probability distribution determined by their covariance $\mathbb{E}(X_i X_j)$ and any positive definite matrix can occur. One can extend this to countably many variables or even to variables indexed by a real parameter if continuity properties of the covariance imply a separable Hilbert space defined by the covariance.

Gaussian processes

To explain Ed's contribution to Euclidean Quantum Field Theory (EQFT), it is useful to first look at path integrals for ordinary quantum mechanics. A finite number of jointly Gaussian random variables have a probability distribution determined by their covariance $\mathbb{E}(X_i X_j)$ and any positive definite matrix can occur. One can extend this to countably many variables or even to variables indexed by a real parameter if continuity properties of the covariance imply a separable Hilbert space defined by the covariance.

Brownian motion is the Gaussian process indexed by $t \geq 0$ with $\mathbb{E}(b(t)b(s)) = \min(t, s)$ and the oscillator process is indexed by $t \in \mathbb{R}$ with $\mathbb{E}(q(t)q(s)) = \exp(-|t - s|)$.

Gaussian processes

To explain Ed's contribution to Euclidean Quantum Field Theory (EQFT), it is useful to first look at path integrals for ordinary quantum mechanics. A finite number of jointly Gaussian random variables have a probability distribution determined by their covariance $\mathbb{E}(X_i X_j)$ and any positive definite matrix can occur. One can extend this to countably many variables or even to variables indexed by a real parameter if continuity properties of the covariance imply a separable Hilbert space defined by the covariance.

Brownian motion is the Gaussian process indexed by $t \geq 0$ with $\mathbb{E}(b(t)b(s)) = \min(t, s)$ and the oscillator process is indexed by $t \in \mathbb{R}$ with $\mathbb{E}(q(t)q(s)) = \exp(-|t - s|)$. We note that the covariances in these cases are fundamental solutions for $-d^2/dt^2$ (with Dirichlet conditions at $t = 0$) and for $-d^2/dt^2 + 1$.

The Feynman-Kac formula

The oscillator process defines a path integral for $H_0 = -\frac{1}{2}\frac{d^2}{dx^2} + \frac{1}{2}x^2 - 1$ in that if Ω_0 is the lowest eigenvector for H_0, then

$$(x\Omega_0, \exp(-tH_0)x\Omega_0) = \mathbb{E}(q(0)q(t))$$

The Feynman-Kac formula

The oscillator process defines a path integral for $H_0 = -\frac{1}{2}\frac{d^2}{dx^2} + \frac{1}{2}x^2 - 1$ in that if Ω_0 is the lowest eigenvector for H_0, then

$$(x\Omega_0, \exp(-tH_0)x\Omega_0) = \mathbb{E}(q(0)q(t))$$

and this leads to a Feynman-Kac formula

$$(\Omega_0, \exp(-t(H_0 + V(x)))\Omega_0) = \mathbb{E}\left(\exp(-\int_0^t V(q(s))ds)\right)$$

EQFT on One Foot

What Ed realized is that the proper path integral for a massive free Bose field is just a Gaussian process with covariance given by the Green's function for $-\Delta + 1$.

EQFT on One Foot

What Ed realized is that the proper path integral for a massive free Bose field is just a Gaussian process with covariance given by the Green's function for $-\Delta + 1$. In two dimensions this Green's function has a logarithmic divergence at 0 so one need to view this field as a distribution.

EQFT on One Foot

What Ed realized is that the proper path integral for a massive free Bose field is just a Gaussian process with covariance given by the Green's function for $-\Delta + 1$. In two dimensions this Green's function has a logarithmic divergence at 0 so one need to view this field as a distribution. The analog of $(x\Omega_0, \exp(-tH_0)x\Omega_0) = \mathbb{E}(q(0)q(t))$ is just

$$(\phi(x_0)\Omega_0, \exp(-tH_0)\phi(x_1)\Omega_0) = \mathbb{E}(\Phi(x_0, 0)\Phi(x_1, t))$$

where ϕ is the quantum field and Φ its "path" which is called the Euclidean field.

EQFT on One Foot

This leads to a Feynman-Kac formula. If $H_\ell = H_0 + \int_0^\ell : \phi(x)^4 : dx$, then

EQFT on One Foot

This leads to a Feynman-Kac formula. If $H_\ell = H_0 + \int_0^\ell : \phi(x)^4 : dx$, then

$$(\Omega_0, \exp(-tH_\ell)\Omega_0) = \mathbb{E}\left(\exp(-\int_0^t \int_0^\ell : \Phi(x,s)^4 : dxds)\right)$$

EQFT on One Foot

This leads to a Feynman-Kac formula. If $H_\ell = H_0 + \int_0^\ell : \phi(x)^4 : dx$, then

$$(\Omega_0, \exp(-tH_\ell)\Omega_0) = \mathbb{E}\left(\exp(-\int_0^t \int_0^\ell : \Phi(x,s)^4 : dxds)\right)$$

In particular, by the Euclidean covariance, one has what is called *Neslon's symmetry*:

$$(\Omega_0, \exp(-tH_\ell)\Omega_0) = (\Omega_0, \exp(-\ell H_t)\Omega_0)$$

EQFT on One Foot

This leads to a Feynman-Kac formula. If $H_\ell = H_0 + \int_0^\ell : \phi(x)^4 : dx$, then

$$(\Omega_0, \exp(-tH_\ell)\Omega_0) = \mathbb{E}\left(\exp(-\int_0^t \int_0^\ell : \Phi(x,s)^4 : dxds)\right)$$

In particular, by the Euclidean covariance, one has what is called *Neslon's symmetry*:

$$(\Omega_0, \exp(-tH_\ell)\Omega_0) = (\Omega_0, \exp(-\ell H_t)\Omega_0)$$

By combining this with what hypercontractivity says about decoupling in the Euclidean space, Ed found a proof of the linear lower bound. originally found by Glimm-Jaffe:

$$E_\ell \geq -C\ell \text{ for some finite } C$$

A Thumbnail History

Between Ed's hypercontractive paper and the end of 1971, there was an intensive effort to try and construct interacting quantum fields by studying the spatial cutoff Hamiltonians and taking some kind of limit.

A Thumbnail History

Between Ed's hypercontractive paper and the end of 1971, there was an intensive effort to try and construct interacting quantum fields by studying the spatial cutoff Hamiltonians and taking some kind of limit. The leaders were Glimm and Jaffe with other contributions by Federbush, Hoegh-Krohn, Rosen, Segal and Simon.

A Thumbnail History

Between Ed's hypercontractive paper and the end of 1971, there was an intensive effort to try and construct interacting quantum fields by studying the spatial cutoff Hamiltonians and taking some kind of limit. The leaders were Glimm and Jaffe with other contributions by Federbush, Hoegh-Krohn, Rosen, Segal and Simon. Two of the high points were the linear lower bound and a technical result known as phi bounds. Both were obtained by Glimm and Jaffe in what we regarded as difficult arguments.

A Thumbnail History

Between Ed's hypercontractive paper and the end of 1971, there was an intensive effort to try and construct interacting quantum fields by studying the spatial cutoff Hamiltonians and taking some kind of limit. The leaders were Glimm and Jaffe with other contributions by Federbush, Hoegh-Krohn, Rosen, Segal and Simon. Two of the high points were the linear lower bound and a technical result known as phi bounds. Both were obtained by Glimm and Jaffe in what we regarded as difficult arguments.

Ed probably found his results by early 1971 and talked about it at Princeton in Feb. or March of that year; he gave some poorly attended follow-up talks which I missed since I was out of town.

A Thumbnail History

Between Ed's hypercontractive paper and the end of 1971, there was an intensive effort to try and construct interacting quantum fields by studying the spatial cutoff Hamiltonians and taking some kind of limit. The leaders were Glimm and Jaffe with other contributions by Federbush, Hoegh-Krohn, Rosen, Segal and Simon. Two of the high points were the linear lower bound and a technical result known as phi bounds. Both were obtained by Glimm and Jaffe in what we regarded as difficult arguments.

Ed probably found his results by early 1971 and talked about it at Princeton in Feb. or March of that year; he gave some poorly attended follow-up talks which I missed since I was out of town. He also talked about the work at a summer conference in Berkeley which I know Glimm and Jaffe attended.

A Thumbnail History

With one exception, no one paid any attention to this work until early January, 1972.

A Thumbnail History

With one exception, no one paid any attention to this work until early January, 1972. Considering the explosion of work in the 18 months after that it is surprising that there was no impact until then. I think some of the reasons were:

A Thumbnail History

With one exception, no one paid any attention to this work until early January, 1972. Considering the explosion of work in the 18 months after that it is surprising that there was no impact until then. I think some of the reasons were:

1. The presentations were almost Delphic and the purpose of his machinery wasn't clear – at least to me.

A Thumbnail History

With one exception, no one paid any attention to this work until early January, 1972. Considering the explosion of work in the 18 months after that it is surprising that there was no impact until then. I think some of the reasons were:

1. The presentations were almost Delphic and the purpose of his machinery wasn't clear – at least to me.
2. Ed emphasized various probabilistic features such as the Markov property and that made it difficult for the many analysts working in the area to understand.

A Thumbnail History

With one exception, no one paid any attention to this work until early January, 1972. Considering the explosion of work in the 18 months after that it is surprising that there was no impact until then. I think some of the reasons were:

1. The presentations were almost Delphic and the purpose of his machinery wasn't clear – at least to me.
2. Ed emphasized various probabilistic features such as the Markov property and that made it difficult for the many analysts working in the area to understand.
3. He had no new results. What should have been striking was his simple new proof of the linear lower bound but shortly before, I'd found my own simple proof so some of us discounted this

A Thumbnail History

With one exception, no one paid any attention to this work until early January, 1972. Considering the explosion of work in the 18 months after that it is surprising that there was no impact until then. I think some of the reasons were:

1. The presentations were almost Delphic and the purpose of his machinery wasn't clear – at least to me.
2. Ed emphasized various probabilistic features such as the Markov property and that made it difficult for the many analysts working in the area to understand.
3. He had no new results. What should have been striking was his simple new proof of the linear lower bound but shortly before, I'd found my own simple proof so some of us discounted this – although in retrospect, Ed's proof was much better than mine

A Thumbnail History

With one exception, no one paid any attention to this work until early January, 1972. Considering the explosion of work in the 18 months after that it is surprising that there was no impact until then. I think some of the reasons were:

1. The presentations were almost Delphic and the purpose of his machinery wasn't clear – at least to me.
2. Ed emphasized various probabilistic features such as the Markov property and that made it difficult for the many analysts working in the area to understand.
3. He had no new results. What should have been striking was his simple new proof of the linear lower bound but shortly before, I'd found my own simple proof so some of us discounted this – although in retrospect, Ed's proof was much better than mine
4. Some of us weren't clever enough to pick up on his brilliant insights!

A Thumbnail History

Francesco Guerra was a young protege of Caianiello, an Italian physicist and good friend of Arthur Wightman. Italian money was found for him to visit Princeton for two years starting in the fall of 1970.

A Thumbnail History

Francesco Guerra was a young protege of Caianiello, an Italian physicist and good friend of Arthur Wightman. Italian money was found for him to visit Princeton for two years starting in the fall of 1970. Those who know the current confident Prof. Guerra would be surprised to find out that he was shy and quiet and he had almost no interactions with anyone but Wightman and Sergio Albeverio.

A Thumbnail History

Francesco Guerra was a young protege of Caianiello, an Italian physicist and good friend of Arthur Wightman. Italian money was found for him to visit Princeton for two years starting in the fall of 1970. Those who know the current confident Prof. Guerra would be surprised to find out that he was shy and quiet and he had almost no interactions with anyone but Wightman and Sergio Albeverio.

In Dec. 1971, Lon Rosen, Wightman and I attended a conference in Miami and Arthur told us that before he left, Guerra had said he had something to report and wanted to meet with us.

A Thumbnail History

Francesco Guerra was a young protege of Caianiello, an Italian physicist and good friend of Arthur Wightman. Italian money was found for him to visit Princeton for two years starting in the fall of 1970. Those who know the current confident Prof. Guerra would be surprised to find out that he was shy and quiet and he had almost no interactions with anyone but Wightman and Sergio Albeverio.

In Dec. 1971, Lon Rosen, Wightman and I attended a conference in Miami and Arthur told us that before he left, Guerra had said he had something to report and wanted to meet with us. I had trouble remembering who Guerra was, but agreed that we should meet with him which we did in Wightman's office after our return.

A Thumbnail History

Guerra began by writing on the blackboard, three things he was going to prove about : Φ^4 : field theories in two space-time dimensions.

A Thumbnail History

Guerra began by writing on the blackboard, three things he was going to prove about : Φ^4 : field theories in two space-time dimensions. It was as if he was from a different planet. For example, it was difficult enough to prove that $E_\ell \geq -C\ell$ for some finite C and he said he could prove that E_ℓ/ℓ had a limit!

A Thumbnail History

Guerra began by writing on the blackboard, three things he was going to prove about : Φ^4 : field theories in two space-time dimensions. It was as if he was from a different planet. For example, it was difficult enough to prove that $E_\ell \geq -C\ell$ for some finite C and he said he could prove that E_ℓ/ℓ had a limit! Years later, afer Lon Rosen and I had a successful collaboration with Guerra, we compared notes on that first meeting and we had the same thought!

A Thumbnail History

Guerra began by writing on the blackboard, three things he was going to prove about : Φ^4 : field theories in two space-time dimensions. It was as if he was from a different planet. For example, it was difficult enough to prove that $E_\ell \geq -C\ell$ for some finite C and he said he could prove that E_ℓ/ℓ had a limit! Years later, afer Lon Rosen and I had a successful collaboration with Guerra, we compared notes on that first meeting and we had the same thought! "Yeah! Sure. You going to prove that".

A Thumbnail History

Guerra began by writing on the blackboard, three things he was going to prove about : Φ^4 : field theories in two space-time dimensions. It was as if he was from a different planet. For example, it was difficult enough to prove that $E_\ell \geq -C\ell$ for some finite C and he said he could prove that E_ℓ/ℓ had a limit! Years later, afer Lon Rosen and I had a successful collaboration with Guerra, we compared notes on that first meeting and we had the same thought! "Yeah! Sure. You going to prove that".

He began by writing down Nelson's symmetry, which we'd seen but not paid attention to.

A Thumbnail History

Guerra began by writing on the blackboard, three things he was going to prove about : Φ^4 : field theories in two space-time dimensions. It was as if he was from a different planet. For example, it was difficult enough to prove that $E_\ell \geq -C\ell$ for some finite C and he said he could prove that E_ℓ/ℓ had a limit! Years later, afer Lon Rosen and I had a successful collaboration with Guerra, we compared notes on that first meeting and we had the same thought! "Yeah! Sure. You going to prove that".

He began by writing down Nelson's symmetry, which we'd seen but not paid attention to. Literally 15 minutes later, he had proven his claims.

A Thumbnail History

While he didn't show it that way, the limit result can be proven by noting that, by the spectral theorem, $\log\left[(\Omega_0, \exp(-tH_\ell)\Omega_0)\right]$ is convex in t and so in ℓ and then by taking a limit as $t \to \infty$, E_ℓ is concave in ℓ which yields the limit.

A Thumbnail History

While he didn't show it that way, the limit result can be proven by noting that, by the spectral theorem, $\log\left[(\Omega_0, \exp(-tH_\ell)\Omega_0)\right]$ is convex in t and so in ℓ and then by taking a limit as $t \to \infty$, E_ℓ is concave in ℓ which yields the limit. We were blown away!

A Thumbnail History

While he didn't show it that way, the limit result can be proven by noting that, by the spectral theorem, $\log[(\Omega_0, \exp(-tH_\ell)\Omega_0)]$ is convex in t and so in ℓ and then by taking a limit as $t \to \infty$, E_ℓ is concave in ℓ which yields the limit. We were blown away!

Guerra invited us to join him in further researches and within a week, we found the next term in the asymptotics of E_ℓ and a simple proof of the ϕ bounds.

A Thumbnail History

While he didn't show it that way, the limit result can be proven by noting that, by the spectral theorem, $\log\left[(\Omega_0, \exp(-tH_\ell)\Omega_0)\right]$ is convex in t and so in ℓ and then by taking a limit as $t \to \infty$, E_ℓ is concave in ℓ which yields the limit. We were blown away!

Guerra invited us to join him in further researches and within a week, we found the next term in the asymptotics of E_ℓ and a simple proof of the ϕ bounds. By coincidence, Glimm gave the math physics seminar the week after that, where he discussed their proof of their ϕ bounds without being able to explain all the details.

A Thumbnail History

While he didn't show it that way, the limit result can be proven by noting that, by the spectral theorem, $\log[(\Omega_0, \exp(-tH_\ell)\Omega_0)]$ is convex in t and so in ℓ and then by taking a limit as $t \to \infty$, E_ℓ is concave in ℓ which yields the limit. We were blown away!

Guerra invited us to join him in further researches and within a week, we found the next term in the asymptotics of E_ℓ and a simple proof of the ϕ bounds. By coincidence, Glimm gave the math physics seminar the week after that, where he discussed their proof of their ϕ bounds without being able to explain all the details. After his talk, the three of us waylaid him with our simple proof.

A Thumbnail History

While he didn't show it that way, the limit result can be proven by noting that, by the spectral theorem, $\log\left[(\Omega_0, \exp(-tH_\ell)\Omega_0)\right]$ is convex in t and so in ℓ and then by taking a limit as $t \to \infty$, E_ℓ is concave in ℓ which yields the limit. We were blown away!

Guerra invited us to join him in further researches and within a week, we found the next term in the asymptotics of E_ℓ and a simple proof of the ϕ bounds. By coincidence, Glimm gave the math physics seminar the week after that, where he discussed their proof of their ϕ bounds without being able to explain all the details. After his talk, the three of us waylaid him with our simple proof. Like Lon and me, he was blown away by the power of Euclidean methods.

A Thumbnail History

While he didn't show it that way, the limit result can be proven by noting that, by the spectral theorem, $\log\left[(\Omega_0, \exp(-tH_\ell)\Omega_0)\right]$ is convex in t and so in ℓ and then by taking a limit as $t \to \infty$, E_ℓ is concave in ℓ which yields the limit. We were blown away!

Guerra invited us to join him in further researches and within a week, we found the next term in the asymptotics of E_ℓ and a simple proof of the ϕ bounds. By coincidence, Glimm gave the math physics seminar the week after that, where he discussed their proof of their ϕ bounds without being able to explain all the details. After his talk, the three of us waylaid him with our simple proof. Like Lon and me, he was blown away by the power of Euclidean methods.

Ed had opened up a new world and essentially all work since has been in the Euclidean realm.

An Innovator

In looking over these two amazing contributions from the perspective of a,

An Innovator

In looking over these two amazing contributions from the perspective of a, er, senior mathematician, I am struck by how innovative they were.

An Innovator

In looking over these two amazing contributions from the perspective of a, er, senior mathematician, I am struck by how innovative they were. Both had at least some prior work: The Bonami-Gross optimal estimate on a two point space is named because Bonami found them, while studying Fourier analysis on groups, in 1970 (before Nelson's optimal estimate but after his hypercontractive paper). And Stam in 1959 and Blachman in 1965 had the results equivalent to the optimal log Sobolev inequalities for Gaussian measures but not in a context of infinitesimal semigroup bounds.

An Innovator

In looking over these two amazing contributions from the perspective of a, er, senior mathematician, I am struck by how innovative they were. Both had at least some prior work: The Bonami-Gross optimal estimate on a two point space is named because Bonami found them, while studying Fourier analysis on groups, in 1970 (before Nelson's optimal estimate but after his hypercontractive paper). And Stam in 1959 and Blachman in 1965 had the results equivalent to the optimal log Sobolev inequalities for Gaussian measures but not in a context of infinitesimal semigroup bounds. Ed's 1966 paper was not only the first result in a subject that was thought to be impossibly hard but was built with several striking tools that had little relation to anything done earlier.

An Innovator

EQFT also had some precursors. Shortly before, Pitt had written down the same objects that Ed did for his free fields – that is the Gaussian process indexed by $\mathbb{R}^n$ with covariance given by the Green's function of $-\Delta + 1$ and he had noted it had the Markov property.

An Innovator

EQFT also had some precursors. Shortly before, Pitt had written down the same objects that Ed did for his free fields – that is the Gaussian process indexed by $\mathbb{R}^n$ with covariance given by the Green's function of $-\Delta + 1$ and he had noted it had the Markov property. But he'd presented as an interesting multiparameter process without realizing the connection to QFT.

An Innovator

EQFT also had some precursors. Shortly before, Pitt had written down the same objects that Ed did for his free fields – that is the Gaussian process indexed by $\mathbb{R}^n$ with covariance given by the Green's function of $-\Delta + 1$ and he had noted it had the Markov property. But he'd presented as an interesting multiparameter process without realizing the connection to QFT.

Nakano and Schwinger had noted the vacuum expectation values had a continuation to imaginary time that had Euclidean invariance in place of Minkowski.

An Innovator

Most importantly, Symanzik had an amazing unpublished 1965 NYU 50 page manuscript with a shorter J. Math. Phys. article and published summer school lecture. He realized that there was an underlying measure space, that there were analogies to statistical mechanics (something Nelson didn't do but Guerra-Rosen-Simon did exploit) and he developed some integral equations.

An Innovator

Most importantly, Symanzik had an amazing unpublished 1965 NYU 50 page manuscript with a shorter J. Math. Phys. article and published summer school lecture. He realized that there was an underlying measure space, that there were analogies to statistical mechanics (something Nelson didn't do but Guerra-Rosen-Simon did exploit) and he developed some integral equations.

Symanzik deserves admiration but Ed still developed related ideas in a form that emphasized the mathematical structures and motivated important follow-up work. Ed was basically inventing a framework from close to scratch that has been the standard one now for over 40 years.

An Unusual Publication Record

You might have expected that revolutionary results like the two I've discussed, especially by a Princeton professor would have wound up in the *Annals of Mathematics* (where Ed did have some other papers)

An Unusual Publication Record

You might have expected that revolutionary results like the two I've discussed, especially by a Princeton professor would have wound up in the *Annals of Mathematics* (where Ed did have some other papers) but not these.

An Unusual Publication Record

You might have expected that revolutionary results like the two I've discussed, especially by a Princeton professor would have wound up in the *Annals of Mathematics* (where Ed did have some other papers) but not these. The hypercontractive result with its three important realizations never appeared in journal but only as a 5 page contribution to a conference proceedings!

An Unusual Publication Record

You might have expected that revolutionary results like the two I've discussed, especially by a Princeton professor would have wound up in the *Annals of Mathematics* (where Ed did have some other papers) but not these. The hypercontractive result with its three important realizations never appeared in journal but only as a 5 page contribution to a conference proceedings!

For EQFT, there were two journal articles - both in the *Journal of Functional Analysis* of 16 and 17 pages respectively.

An Unusual Publication Record

You might have expected that revolutionary results like the two I've discussed, especially by a Princeton professor would have wound up in the *Annals of Mathematics* (where Ed did have some other papers) but not these. The hypercontractive result with its three important realizations never appeared in journal but only as a 5 page contribution to a conference proceedings!

For EQFT, there were two journal articles - both in the *Journal of Functional Analysis* of 16 and 17 pages respectively. The first dealt exclusively with the question of how to go back from EQFT to a Minkowski field – an important issue to justify the usefulness of these ideas although later supplanted by results of Osterwalder-Schrader.

An Unusual Publication Record

The other called *The Free Markoff Field* had his best hypercontractive bound and discussed the free field.

An Unusual Publication Record

The other called *The Free Markoff Field* had his best hypercontractive bound and discussed the free field. But no mention of a Feynman-Kac formula or of his proof of the linear lower bound (which he had put into half a page in the Proceedings of the Berkeley conference).

An Unusual Publication Record

The other called *The Free Markoff Field* had his best hypercontractive bound and discussed the free field. But no mention of a Feynman-Kac formula or of his proof of the linear lower bound (which he had put into half a page in the Proceedings of the Berkeley conference). Guerra's note and the GRS paper on the ϕ bounds are in the reference list for the this paper but are not quoted in the text!

An Unusual Publication Record

The other called *The Free Markoff Field* had his best hypercontractive bound and discussed the free field. But no mention of a Feynman-Kac formula or of his proof of the linear lower bound (which he had put into half a page in the Proceedings of the Berkeley conference). Guerra's note and the GRS paper on the ϕ bounds are in the reference list for the this paper but are not quoted in the text!

It isn't as if the *Annals* was inhospitable to to EQFT. Shortly after this they published a paper of the Guerra-Rosen-Simon on *The* $P(\phi)_2$ *Euclidean quantum field theory as classical statistical mechanics* which was 147 pages.

An Unusual Publication Record

The other called *The Free Markoff Field* had his best hypercontractive bound and discussed the free field. But no mention of a Feynman-Kac formula or of his proof of the linear lower bound (which he had put into half a page in the Proceedings of the Berkeley conference). Guerra's note and the GRS paper on the ϕ bounds are in the reference list for the this paper but are not quoted in the text!

It isn't as if the *Annals* was inhospitable to to EQFT. Shortly after this they published a paper of the Guerra-Rosen-Simon on *The* $P(\phi)_2$ *Euclidean quantum field theory as classical statistical mechanics* which was 147 pages. The only difficulty we had was the paper was so long, we were made to split it in two so it wouldn't take up one whole issue.

An Unusual Publication Record

The other called *The Free Markoff Field* had his best hypercontractive bound and discussed the free field. But no mention of a Feynman-Kac formula or of his proof of the linear lower bound (which he had put into half a page in the Proceedings of the Berkeley conference). Guerra's note and the GRS paper on the ϕ bounds are in the reference list for the this paper but are not quoted in the text!

It isn't as if the *Annals* was inhospitable to to EQFT. Shortly after this they published a paper of the Guerra-Rosen-Simon on *The $P(\phi)_2$ Euclidean quantum field theory as classical statistical mechanics* which was 147 pages. The only difficulty we had was the paper was so long, we were made to split it in two so it wouldn't take up one whole issue. And it had a subsection on the "Feynman-Kac-Nelson" formula.

An Unusual Publication Record

There is one other curiosity about Ed's publication record that I wish to mention. The set of mathematicians can be studied using what for lack of a better term I'll call the Erdős topology where coauthors are joined by a line.

An Unusual Publication Record

There is one other curiosity about Ed's publication record that I wish to mention. The set of mathematicians can be studied using what for lack of a better term I'll call the Erdős topology where coauthors are joined by a line. There is one huge connected component (hence Erdős number) and a few rare singlets of authors without any coauthored papers.

An Unusual Publication Record

There is one other curiosity about Ed's publication record that I wish to mention. The set of mathematicians can be studied using what for lack of a better term I'll call the Erdős topology where coauthors are joined by a line. There is one huge connected component (hence Erdős number) and a few rare singlets of authors without any coauthored papers.

Assuming one ignores papers like the *Notices* memorial for Irving Segal, Ed's thesis advisor, Ed is a member of a triplet – he has a single coauthored paper with W. Stinespring, whose coauthors are Ed and David Shale (whose only joint papers are with Stinespring)!

The Tale of Nelson's Commutator Theorem

Ed was both a gentle man and a gentleman as is seen in the following story.

The Tale of Nelson's Commutator Theorem

Ed was both a gentle man and a gentleman as is seen in the following story. In their work on self-adjoint of time averaged fields, Glimm and Jaffe proved a self-adjointness result for non-semibounded operators, A.

The Tale of Nelson's Commutator Theorem

Ed was both a gentle man and a gentleman as is seen in the following story. In their work on self-adjoint of time averaged fields, Glimm and Jaffe proved a self-adjointness result for non-semibounded operators, A. They required an auxiliary positive operator, N, and needed control over A, [N,A] and [N,[N,A]] in terms of N.

The Tale of Nelson's Commutator Theorem

Ed was both a gentle man and a gentleman as is seen in the following story. In their work on self-adjoint of time averaged fields, Glimm and Jaffe proved a self-adjointness result for non-semibounded operators, A. They required an auxiliary positive operator, N, and needed control over A, [N,A] and [N,[N,A]] in terms of N. Ed found a different method of proof that, more important, only needed control of the commutator and not also the double commutator.

The Tale of Nelson's Commutator Theorem

Ed was both a gentle man and a gentleman as is seen in the following story. In their work on self-adjoint of time averaged fields, Glimm and Jaffe proved a self-adjointness result for non-semibounded operators, A. They required an auxiliary positive operator, N, and needed control over A, [N,A] and [N,[N,A]] in terms of N. Ed found a different method of proof that, more important, only needed control of the commutator and not also the double commutator.

Then Faris and Lavine found an elegant way of using this result to get self-adjointness of Stark Hamiltonians (constant electric field) with fairly general two body potentials.

The Tale of Nelson's Commutator Theorem

Ed was both a gentle man and a gentleman as is seen in the following story. In their work on self-adjoint of time averaged fields, Glimm and Jaffe proved a self-adjointness result for non-semibounded operators, A. They required an auxiliary positive operator, N, and needed control over A, [N,A] and [N,[N,A]] in terms of N. Ed found a different method of proof that, more important, only needed control of the commutator and not also the double commutator.

Then Faris and Lavine found an elegant way of using this result to get self-adjointness of Stark Hamiltonians (constant electric field) with fairly general two body potentials. They could control single commutators but doubly commutators would have required two body potentials to be smooth, so Ed's version was essential.

The Tale of Nelson's Commutator Theorem

Shortly afterwards, Reed and I were polishing Volume 2 of our series of 4 books, *Methods of Modern Mathematical Physics*, and it was natural to explain this method and its application to Stark Hamiltonains. We called the added section *Nelson's Commutator Theorem*.

The Tale of Nelson's Commutator Theorem

Shortly afterwards, Reed and I were polishing Volume 2 of our series of 4 books, *Methods of Modern Mathematical Physics*, and it was natural to explain this method and its application to Stark Hamiltonains. We called the added section *Nelson's Commutator Theorem*. My relations with Glimm and Jaffe were strained over reactions to our work on constructive field theory and they were not happy about this section in Reed-Simon 2. Glimm, in particular, made his displeasure clear several times, so much so that Mike Reed and I sought Arthur Wightman's advice.

The Tale of Nelson's Commutator Theorem

Shortly afterwards, Reed and I were polishing Volume 2 of our series of 4 books, *Methods of Modern Mathematical Physics*, and it was natural to explain this method and its application to Stark Hamiltonains. We called the added section *Nelson's Commutator Theorem*. My relations with Glimm and Jaffe were strained over reactions to our work on constructive field theory and they were not happy about this section in Reed-Simon 2. Glimm, in particular, made his displeasure clear several times, so much so that Mike Reed and I sought Arthur Wightman's advice.

We decided to rename the section *The Commutator Theorem*.

The Tale of Nelson's Commutator Theorem

Shortly afterwards, Reed and I were polishing Volume 2 of our series of 4 books, *Methods of Modern Mathematical Physics*, and it was natural to explain this method and its application to Stark Hamiltonains. We called the added section *Nelson's Commutator Theorem*. My relations with Glimm and Jaffe were strained over reactions to our work on constructive field theory and they were not happy about this section in Reed-Simon 2. Glimm, in particular, made his displeasure clear several times, so much so that Mike Reed and I sought Arthur Wightman's advice.

We decided to rename the section *The Commutator Theorem*. But before we let the publisher know we decided we needed to make sure Ed was OK with this change.

The Tale of Nelson's Commutator Theorem

I explained the situation to Ed and asked if he had any objection to the name change.

The Tale of Nelson's Commutator Theorem

I explained the situation to Ed and asked if he had any objection to the name change. He smiled in his inimitable way and said:

The Tale of Nelson's Commutator Theorem

I explained the situation to Ed and asked if he had any objection to the name change. He smiled in his inimitable way and said: "Of course you can make the change.

The Tale of Nelson's Commutator Theorem

I explained the situation to Ed and asked if he had any objection to the name change. He smiled in his inimitable way and said: "Of course you can make the change. But if you'd prefer, you can keep it and tell them that you wanted to make the change but I wouldn't let you do it."

The Tale of Nelson's Commutator Theorem

I explained the situation to Ed and asked if he had any objection to the name change. He smiled in his inimitable way and said: "Of course you can make the change. But if you'd prefer, you can keep it and tell them that you wanted to make the change but I wouldn't let you do it." !!!!

The Tale of Nelson's Commutator Theorem

I explained the situation to Ed and asked if he had any objection to the name change. He smiled in his inimitable way and said: "Of course you can make the change. But if you'd prefer, you can keep it and tell them that you wanted to make the change but I wouldn't let you do it." !!!!

I thanked him but said it might make his name mud but it would help me.

The Tale of Nelson's Commutator Theorem

I explained the situation to Ed and asked if he had any objection to the name change. He smiled in his inimitable way and said: "Of course you can make the change. But if you'd prefer, you can keep it and tell them that you wanted to make the change but I wouldn't let you do it." !!!!

I thanked him but said it might make his name mud but it would help me.

So the first 1000 copies of our book have a different section title from the one you can buy today on Aamzon!

Loewner and Nelson

I have long been an admirer of the work of Charles Loewner, who like Ed had two great contributions (matrix monotone functions and Loewner evolution).

Loewner and Nelson

I have long been an admirer of the work of Charles Loewner, who like Ed had two great contributions (matrix monotone functions and Loewner evolution). When preparing my talk for Ed's 70+ birthday celebration, I came across words that Bers wrote about his advisor, Loewner, that so fit Ed, I'd like to end with them as I did in that talk:

Loewner and Nelson

a man whom everyone liked, perhaps because he was a man at peace with himself. He conducted a life-long passionate love affair with mathematics, but was neither competitive, nor jealous, nor vain. His kindness and generosity in scientific matters, to students and colleagues alike, were proverbial.

Loewner and Nelson

a man whom everyone liked, perhaps because he was a man at peace with himself. He conducted a life-long passionate love affair with mathematics, but was neither competitive, nor jealous, nor vain. His kindness and generosity in scientific matters, to students and colleagues alike, were proverbial. He seemed to be incapable of malice. His manners were mild and even diffident, but those hid a will of steel. .. , But first and foremost, he was a mathematician.

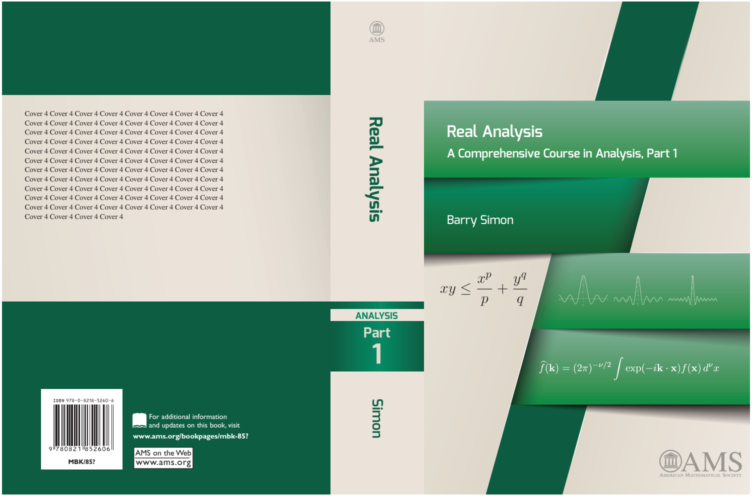
Loewner and Nelson

> *a man whom everyone liked, perhaps because he was a man at peace with himself. He conducted a life-long passionate love affair with mathematics, but was neither competitive, nor jealous, nor vain. His kindness and generosity in scientific matters, to students and colleagues alike, were proverbial. He seemed to be incapable of malice. His manners were mild and even diffident, but those hid a will of steel. .. , But first and foremost, he was a mathematician.*

Contemplate how many really first-class mathematicians about whom one can say they are "neither competitive, nor jealous, nor vain" and appreciate Ed for who he was!

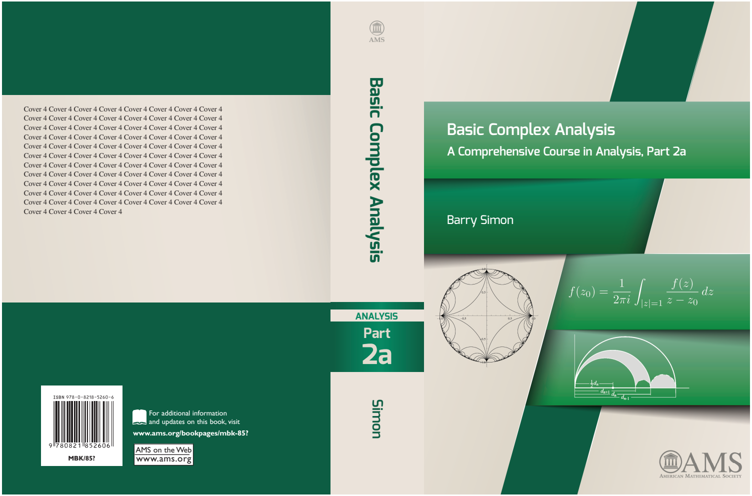
And Now a Word from Our Sponsor

And Now a Word from Our Sponsor



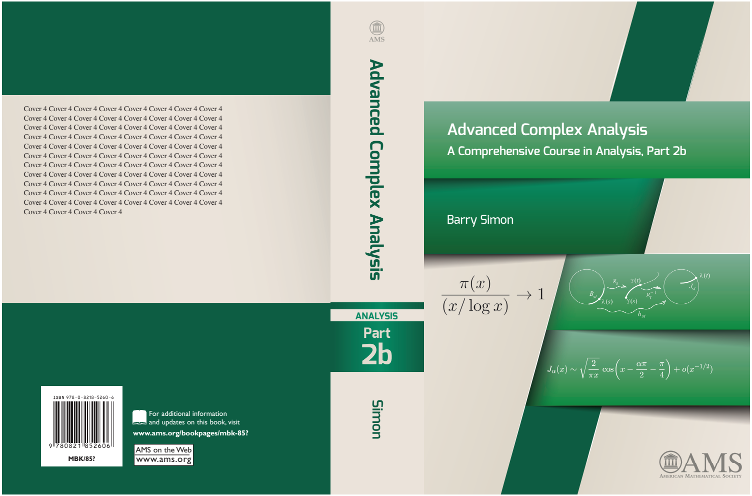
Google *simon comprehensive course preview*

And Now a Word from Our Sponsor



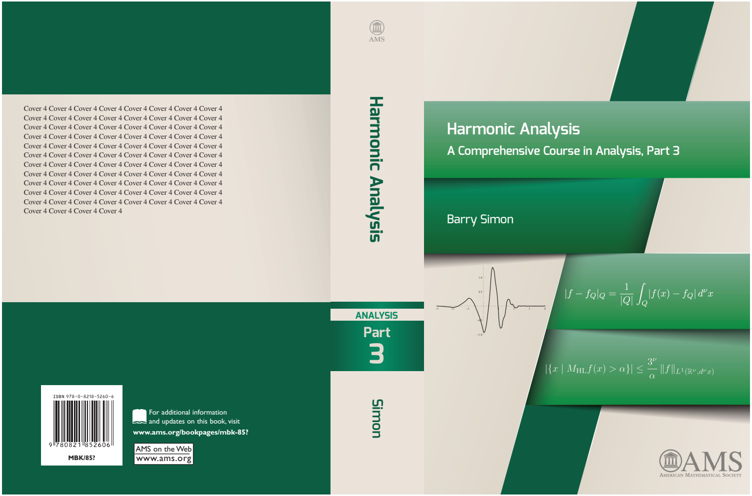
Google *simon comprehensive course preview*

And Now a Word from Our Sponsor



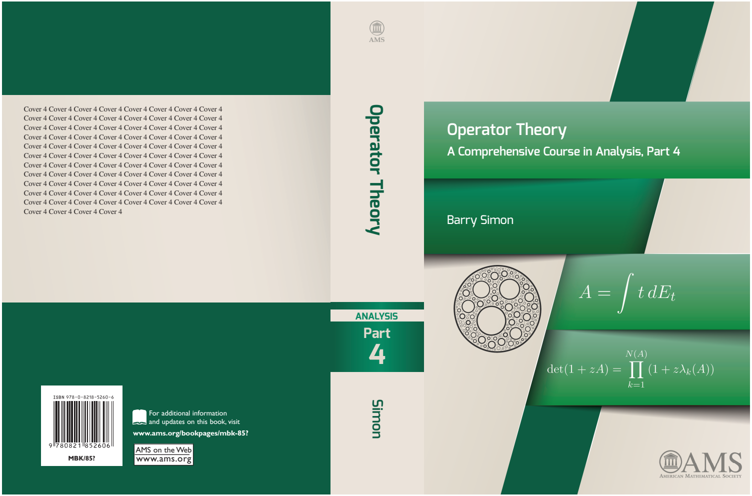
Google *simon comprehensive course preview*

And Now a Word from Our Sponsor



Google *simon comprehensive course preview*

And Now a Word from Our Sponsor



Google *simon comprehensive course preview*