Page VI, line 16: Add "n odd" in the parenthesis.

Page 3, lines 1,2: The assumption that the $F_n$ are disjoint is not used in this proof.

Page 3, line 4: One can also replace “Since ... that” by “Since $X$ is standard Borel.”.

Page 27, Proposition 7.7: Christian Rosendal pointed out the following simpler proof of this proposition, which avoids the need for the Birkhoff ergodic theorem and the assumption that $\mu$ is invariant, by replacing the first half of the proof of Proposition 7.7 with the first half of the proof of Theorem 7.5.

Define $A_m, B_m \subseteq X$ exactly as in the proof of Theorem 7.5, and fix $m \in \mathbb{N}$ such that $\mu(B_m) + \mu(X \setminus A) < \epsilon$. Put $A'' = A_m$, and proceed as before: For each $x \in A''$, let $\ell''(x) > 0$ be the least natural number such that $T^{\ell''(x)}(x) \in A''$, set $k_0(x) = -n$, and recursively define $k_{i+1}(x)$ to be the least natural number such that $T^{k_{i+1}(x)}(x) \in A$ and $k_i(x) + n \leq k_{i+1}(x) \leq \ell''(x) - n$, if such a number exists. Define $B \subseteq X$ by

$$B = \{T^{k_i(x)}(x) : i > 0, x \in A'', \text{ and } k_i(x) \text{ is defined}\},$$

and note that $B \subseteq A$ and $\{T^i(B)\}_{i<n}$ is a pairwise disjoint family which covers $X \setminus (B_m \cup (X \setminus A))$, which is of measure $> 1 - \epsilon$.

Page 28, Remark 7.9: While this follows directly from Proposition 7.7 in the case that $\mu$ is invariant, it is false in general. Given $0 < \delta < \epsilon < 0.25$ and a natural number $n \geq 2$, there is an aperiodic Borel automorphism $T : X \to X$,
a $T$-quasi-invariant probability measure $\mu$ on $X$, and a Borel set $A \subseteq X$ of measure $1 - \delta$ which does not contain an $(\epsilon, n)$-Rokhlin set of measure $\leq 1/n$. To see this, fix an aperiodic Borel automorphism $T' : X' \to X'$ which admits an invariant probability measure $\mu'$, set $X = \{(x, i) : x \in X' \text{ and } i < n\}$, define $T : X \to X$ by

$$T(x, i) = \begin{cases} (x, i + 1) & \text{if } i < n - 1, \\ (T'(x), 0) & \text{otherwise,} \end{cases}$$

and define $\mu$ on $X$ by

$$\mu(B) = (1 - \delta)\mu'(\text{proj}_{X'}(B \cap X_0)) + \sum_{1 \leq i < n} \left( \frac{\delta}{n - 1} \right) \mu'(\text{proj}_{X'}(B \cap X_i)),$$

where $X_i = X' \times \{i\}$. Now suppose, towards a contradiction, that there is an $(\epsilon, n)$-Rokhlin set $B \subseteq X \times \{0\}$ of measure $\leq 1/n$. Then

$$\mu(B) \leq 1/n \text{ and } \sum_{i < n} \mu(T^i(B)) > 1 - \epsilon.$$

It follows from the definition of $\mu$ that for $1 \leq i < n$,

$$\mu(T^i(B)) = \mu(B) \left( \frac{\delta}{n - 1} \right) \left( \frac{1}{1 - \delta} \right),$$

thus

$$\mu(B) \left( 1 + \frac{\delta}{1 - \delta} \right) > 1 - \epsilon.$$

It then follows that

$$\frac{1}{n(1 - \delta)} > (1 - \epsilon),$$

so $2 \leq n < 1/(1 - \delta)(1 - \epsilon)$, which is impossible, since $\delta < \epsilon < 0.25$.

It should be noted, however, that if we replace the requirement that $\mu(A) > 1 - \epsilon$ with the stronger hypothesis that

$$\mu \left( \bigcap_{i < n} T^{-i}(A) \right) > 1 - \epsilon,$$

then $A$ does contain an $(\epsilon, n)$-Rokhlin set of measure $\leq 1/n$. To see this, set

$$\delta = \epsilon - \mu \left( X \setminus \bigcap_{i < n} T^{-i}(A) \right),$$

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appeal to Theorem 7.5 to find a $((\delta, n))$-Rokhlin set $B' \subseteq X$ of measure $\leq 1/n$, and observe that the set $B = A \cap B'$ is as desired.

Page 45, line 6: $\bigcup_{n \in \mathbb{N}} \cap_{m>n} \to \bigcap_{n \in \mathbb{N}} \bigcup_{m>n}$.

Page 45, line 5 of the proof of 10.5: Open parentheses after “$\forall x \in \text{dom}(F_n)$”.

Page 48, line 16-: Add after “identity)”, “such that $f_n(x)E_x, \forall x \in S_n$”.

Page 50, line 14-: $A$ should also contain 1.

Page 50, line 4: In the definition of $f_{\alpha_n}$, $\alpha_n$ should be $\alpha(n)$.

Page 50, line 2 of Theorem 12.1: Add $X$ after “space”.

Page 52, line 6-: Replace $C$ by $X_0 \cup \{x: (x, x) \in F_{\infty}^\alpha\}$; after “$F = E|X_0 \cup F_{\infty}$” add: “to conclude that $\mu(A) = 0$ and thus, as $A$ is a complete section of $C$, $\mu(C) = 0$.”

Page 62, proof of 18.3: Julien Melleray pointed out that one can use the argument in the last paragraph of that proof to show that, for $\mu \in M_f$, we have that $C_{\mu}(E) < r$ holds iff

$$\exists \epsilon \in \mathbb{Q}^+ \forall S \text{ finite } \subseteq \mathbb{N} \exists T \text{ finite } \subseteq \mathbb{N} [C_{\mu}(\Theta_T \cup \{\theta_i|D(\theta_i, \Theta_T)\}_{i \in S}) \leq r - \epsilon].$$

which directly shows that this condition is Borel on $M_f$.

Page 84, line 7: Replace “$x \in F$” by “$xFy$”.

Page 89, line 17: After “where” add “$\bar{A}_0^\theta = A_\theta$ and”.

Page 100, line 7: The first $A_n^\theta$ should be $A_{n+1}^\theta$.

Page 100, line 11: $A_{i+1}^n$ should be $A_{i+1}^{n+1}$.

Page 102, lines 4 and 5: The exponent of $\varphi_\infty$ should be $n_0$ in both cases, not $n$.

Page 102, line 24: The second $\pi_{n,m}$ should be $\pi_{n,m}(\theta)$.

Page 102, line 14-: $\{\varphi_k\}_{k \in K}$ should be $\{\varphi_k\}_{k \in K}$.

Page 102, line 2-: $\psi_i$ should be $\tilde{\psi}_i$.

Page 103, line 3: “extend $\bar{\varphi}_0$” should be “extend $\bar{\varphi}$”.

Page 106, line 6-: Replace “$E|S_E$” by “$F_\epsilon|S_E$”.

Page 108, line 2-: 18.5 should be 18.6.
Page 110, line 2-: \((g \cdot x, h \cdot x)\) should be \((g^{-1} \cdot x, h^{-1} \cdot x)\).

Page 115: Damien Gaboriau has pointed out still another way of seeing that the cost of any infinite amenable group is 1. Suppose, towards a contradiction, that such a group \(\Gamma\) acts freely on a standard Borel space \(X\) in a Borel way with invariant ergodic probability measure \(\mu\), and \(C_\mu(E^X_\Gamma) > 1\). By Lemma 28.12, there is a Borel subtreesing \(T \subseteq E^X_\Gamma\) generating an ergodic equivalence relation \(E_T\) of cost strictly greater than 1. Since subequivalence relations of \(\mu\)-amenable equivalence relations are \(\mu\)-amenable, it follows that \(E_T\) is \(\mu\)-amenable. So from [JKL, 3.23] (which generalizes a result in [A1]), we have that almost every component of \(T\) has at most 2 ends, from which it follows (see, e.g., [JKL, 3.19]) that \(E_T\) is hyperfinite a.e., so has cost 1, a contradiction.

Page 115: After 31.1 add:

Part i) follows from 9.2 and 10.2 and a proof of part ii) is essentially contained in Example 9.4.

Page 121, 35.5: Ioana has extended this result by weakening normality to almost normality and dropping the assumption that \(N\) has fixed price.

Pages 123 and 128: Problem 35.7 has been solved by Abert and Nikolov. The answer is negative. See: M. Abert and N. Nikolov, Rank gradient, cost of groups and the rank versus Heegard genus problem, arXiv:math/0701361v3.