

**HOMEWORK 5**

1. This question gives two ways to compute  $\pi_1(\mathbb{R}P^n)$ .
  - (a) Use the definition of  $\mathbb{R}P^n$  as a quotient of  $S^n$  to compute  $\pi_1(\mathbb{R}P^n)$ .
  - (b) Construct a cell decomposition for  $\mathbb{R}P^n$  and use it to give a presentation for  $\pi_1(\mathbb{R}P^n)$ . Check that your presentation agrees with your answer from part (a).
2. Let  $X$  be the 2-sphere with the north and south poles identified. Write down and simplify a presentation for  $\pi_1(X)$ .
3. Let  $T^2$  be the 2-torus. Put a cell structure on  $T^2$ , with one 2-cell, two 1-cells and one 0-cell. Describe a cell structure on  $\mathbb{R}^2$  that is preserved by the usual action of  $\mathbb{Z}^2$ .
4. Prove that the composition of two covering maps is a covering map.
5.
  - (a) Write down a presentation for the fundamental group of the Klein bottle.
  - (b) Describe a covering map from the 2-torus to the Klein bottle.
  - (c) Deduce that the fundamental group of the Klein bottle acts freely discontinuously on  $\mathbb{R}^2$ .