

**HOMEWORK 6**

1. Let  $C$  be a chain complex in which  $C_i \cong \mathbb{Z}^{a_i}$  for each  $i \geq 0$ . Let  $b_i$  be the  $i$ th Betti number of  $C$ . Assume that  $a_i = 0$  for all but finitely many  $i$ . Prove that

$$\sum_i (-1)^i a_i = \sum_i (-1)^i b_i.$$

(Note that this was set as Exercise 5.3 in the notes.)

2. Draw an explicit triangulation of the 2-dimensional torus  $T$ . Compute the homology of the corresponding chain complex *with coefficients in*  $\mathbb{R}$ .
3. Let  $K$  be a simplicial polyhedron, let  $C_i$  be the group of  $i$ -chains and let  $\partial_i : C_i \rightarrow C_{i-1}$  be as defined in class. Prove that  $\partial_{i-1} \circ \partial_i = 0$  for all  $i$ . (Note that this was set as Exercise 5.9 in the notes.)
4. Let  $K$  be a 1-dimensional simplicial polyhedron. Compute the simplicial homology of  $K$ .
5. Let  $K$  be a simplicial polyhedron such that  $|K|$  is homeomorphic to  $S^1$ . How many simplicial maps are there from  $K$  to itself?