

HOMEWORK 7

1. Construct a triangulation of $\mathbb{R}P^2$.
2. (a) Find an embedding of the 1-skeleton of a 3-dimensional simplex as a simplicial polyhedron in \mathbb{R}^2 .
(b) Prove that the 1-skeleton of a 4-dimensional simplex cannot be embedded as a simplicial polyhedron in \mathbb{R}^2 .
3. Let $f, g : C_i \rightarrow C'_i$ be homomorphisms of chain complexes. Recall that a *chain homotopy* is a set of homomorphisms $h : C_i \rightarrow C'_{i+1}$ such that

$$g(c) - f(c) = \partial'_{i=1} h(c) + h \partial_i(c)$$

for each $c \in C_i$. Prove that f and g induce the same maps on homology.

4. (a) Let X be a graph (ie a 1-dimensional CW-complex). Explain how to triangulate X by subdividing each edge.
(b) Calculate the singular homology groups of X .
Hint: In an earlier exercise, you calculated the simplicial homology of a simplicial graph.
5. Calculate the singular homology groups of the punctured torus (ie a torus with a point removed).